Unit Vectors in 3-D

Not surprisingly, these are just like unit vectors in 2-D. Once again, a unit vector has a length or one unit.

The "special" unit vectors are:

$$i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad j = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad k = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

We can write any column vector in i, j, k form. For example:

$$\mathbf{a} = \begin{pmatrix} 11 \\ 19 \\ 7 \end{pmatrix} \text{ can be written as } 11i + 19j + 7k$$

Finding a unit vector in the same direction as a given vector can be done in the same way we did 2-D unit vectors.

Let us find the unit vector in the direction of:

$$i + 2j + k$$

We will need to find the length [magnitude] of this vector.

Length = $\sqrt{1^2 + 2^2 + 1^2}$

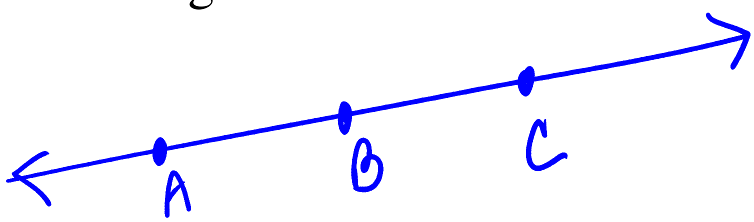
$= \sqrt{6}$

So the unit vector in the same direction is:

$\frac{1}{\sqrt{6}}(i+2j+k)$ OR $\frac{1}{\sqrt{6}}\begin{pmatrix}1\\2\\1\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{6}}\\ \frac{2}{\sqrt{6}}\\ \frac{1}{\sqrt{6}}\end{pmatrix}$

Collinear points and ratio of division

Three or more points are said to be collinear if they lie on the same straight line.



A, B, and C are collinear if $\vec{AB} = k\,\vec{BC}$ where $k \neq 0$

Let us look at Example 13 on page 391

Example 13

Prove that A(−1, 2, 3), B(4, 0, −1) and C(14, −4, −9) are collinear and hence find the ratio in which B divides CA.

$$\overrightarrow{AB} = \begin{bmatrix}5\\-2\\-4\end{bmatrix} \qquad \overrightarrow{BC} = \begin{bmatrix}10\\-4\\-8\end{bmatrix} = 2\begin{bmatrix}5\\-2\\-4\end{bmatrix} \qquad \therefore\ \overrightarrow{BC} = 2\overrightarrow{AB}$$

$\therefore$ BC is parallel to AB and since B is common to both, A, B and C are collinear.

To find the ratio in which B divides CA, we find

$$\overrightarrow{CB} : \overrightarrow{BA} = -2\begin{bmatrix}5\\-2\\-4\end{bmatrix} : -\begin{bmatrix}5\\-2\\-4\end{bmatrix} = 2:1$$

$\therefore$ B divides CA internally in the ratio 2 : 1.

So what are our basic steps to prove that points are collinear? FIND $\vec{AB}$, $\vec{BC}$

SHOW $\vec{BC} = k(\vec{AB})$

Need TO SHARE A point

And now for the last of the 3-D rules!!!

If $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, then $a_1 b_1 + a_2 b_2 + a_3 b_3$

So you can see that the dot [or scalar] product is the same in 3-D as it was in 2-D.

The same properties hold for 3-D. Namely, we can use the dot product to find angles or to see if two vectors are perpendicular.

- ▶ $\mathbf{a} \bullet \mathbf{b} = \mathbf{b} \bullet \mathbf{a}$
- ▶ $\mathbf{a} \bullet \mathbf{a} = |\mathbf{a}|^2$
- ▶ $\mathbf{a} \bullet (\mathbf{b} + \mathbf{c}) = \mathbf{a} \bullet \mathbf{b} + \mathbf{a} \bullet \mathbf{c}$ and
  $(\mathbf{a} + \mathbf{b}) \bullet (\mathbf{c} + \mathbf{d}) = \mathbf{a} \bullet \mathbf{c} + \mathbf{a} \bullet \mathbf{d} + \mathbf{b} \bullet \mathbf{c} + \mathbf{b} \bullet \mathbf{d}$

From page 392

Let us look at some interesting problems on page 391.

4. Find t if $\underbrace{\begin{pmatrix} 3 \\ -1 \\ t \end{pmatrix}}_{a}$ and $\underbrace{\begin{pmatrix} 2t \\ -3 \\ -4 \end{pmatrix}}_{b}$ are perpendicular.

That must mean that $a \bullet b = 0$

$$a \bullet b = \begin{pmatrix} 3 \\ -1 \\ t \end{pmatrix} \bullet \begin{pmatrix} 2t \\ -3 \\ -4 \end{pmatrix}$$

$$0 = (3)(2t) + (-1)(-3) + (t)(-4)$$

$$0 = 6t + 3 - 4t$$

$$-3 = 2t$$

$$\tfrac{-3}{2} = t$$

#5 Show that the following vectors are mutually perpendicular

$$a = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \qquad b = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \qquad c = \begin{pmatrix} 1 \\ 5 \\ -4 \end{pmatrix}$$

What do we need to do?

$a \cdot b$; $b \cdot c$; $a \cdot c$

$a \cdot b = (-3) + 1 + 2 = 0$

$b \cdot c = -1 + 5 - 4 = 0$

$a \cdot c = 3 + 5 - 8 = 0$

✓ pg 395

Homework: Review 16A all [if you have difficulty with #8, then look at Example 15 on page 394]

Write these up neatly so we can have a presentation of all of the solutions!