

Are You Ready for Calculus?

A great review of concepts necessary for calculus

I) Simplify the following fractions:

$$1) \frac{1}{x} + \frac{1}{y}$$

$$2) \frac{1}{x} + \frac{1}{x^2}$$

$$3) \frac{\frac{1}{x} + 1}{\frac{1}{x}}$$

$$4) \frac{\frac{x}{x+y}}{x} =$$

$$5) \frac{3x}{3x+y}$$

$$6) \frac{\frac{1}{x+h} + \frac{1}{x}}{x}$$

II) Factor each expression:

$$7. x^2 - 16$$

$$8) x^2 - x - 6$$

$$9) 6x^2 - x - 2$$

$$10) 4x^3 - 19x^2 - 5x$$

$$11) x^2 + 9$$

$$12) x^4 - 13x^2 - 30$$

$$13) x^3 + 27$$

$$14) x^3 - 8$$

$$15) (2x-3)^3(x+1) + (x-3)(2x-3)^2$$

$$16) (3x-2)^4(x+3) + (x+3)^2(3x-2)^3$$

III) Solve the following equations for x:

$$17) x^2 + 5x + 24 = 0$$

$$18) x^2 - 9 = 5$$

$$19) x^3 - x^2 - x = 0$$

$$20) 3x - 5x - 2 = 0$$

$$21) x^2 - 4x = 0$$

$$22) (x-1)(x^2 - 11x + 30) = 0$$

$$23) \sqrt{x} + 1 = 41$$

$$24) \frac{y}{x+1} = \frac{z}{x}$$

$$25) \sqrt[3]{x+1} - 4 = -1$$

$$26) \log_5 x = 2$$

$$27) x^{-2} = \frac{1}{9}$$

$$28) \log_3 \frac{1}{81} = x$$

$$29) 2\sqrt{x} = x - 3$$

$$30) \log_8 x = \frac{-1}{3}$$

$$31) \frac{8+x}{x} - 5 = 0$$

$$32) x^{-1} = -3$$

$$33) x^{\frac{4}{3}} = 81$$

$$34) x^2 + x - 1 = 0$$

$$35) \sqrt{x} = \sqrt[3]{x}$$

$$36) (x-3)^2 + 9 = 25$$

$$37) 3x^2 - 6x - 24 = 0$$

III) Solve the following equations **for x** continued:

$$38) \frac{2x-1}{(x+2)(x^2+3)} = 0$$

$$39) x^3 - 2x^2 - 5x + 6 = 0$$

$$40) \frac{x}{x+2} - \frac{2}{2x-1} = \frac{1}{5}$$

$$41) xy - z = z + wx - 3x$$

$$42) \frac{4x}{x^2+3} - 1 = 0$$

$$43) (2x-1)^2(x-5)^2 + (2x-1)^3(x-5) = 0$$

IV. Show that each equation is true by simplifying the following:

$$44) \frac{1}{\sqrt{x} + \sqrt{y}} = \frac{\sqrt{x} - \sqrt{y}}{x - y}$$

$$45) \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$46) \frac{x-y}{\sqrt{x} + \sqrt{y}} = \sqrt{x} - \sqrt{y}$$

$$47) \frac{\sqrt{x+4} - 2}{x} = \frac{1}{\sqrt{x+4} + 2}$$

V. Which of the following expressions equals (is the same as) $\log 4$?

$$48) 2\log 2$$

$$49) \frac{\log 8}{\log 2}$$

$$50) \log 8 - \log 2$$

$$51) \log 4 + \log 1$$

$$52) \log 4 \cdot \log 1$$

$$53) (\log 2)^2$$

VI) Write an equation of a line based on the given information:

54) Find the equation of a line that has a slope of 5 and passes through the point (3, -4).

55) Find the equation of the line that passes through the points (4, 1) and (3, -2).

56) Find the equation of the line that passes through the points (-2, 1) and is parallel to the line $4x + 2y = -1$.

57) Find the equation of the line that has a slope of 0 and passes through the point (-5, 1).

58) Find the equation of the line that passes through the origin and is perpendicular to the line $3x + 4y = -7$.

59) Find the equation of the line that has an undefined slope and passes through the point (4, -5).

60) Find the equation of the line that has an x-intercept of 5 and a y-intercept of 3.

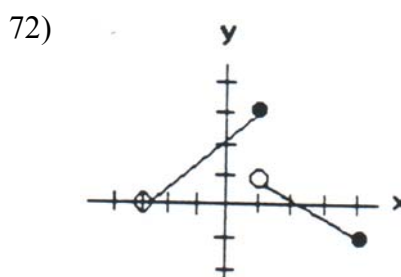
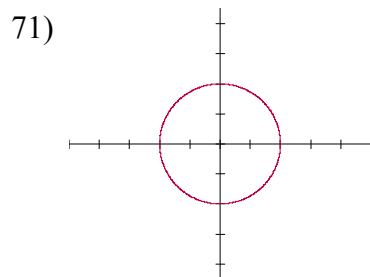
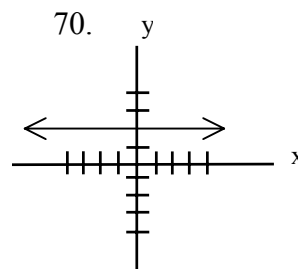
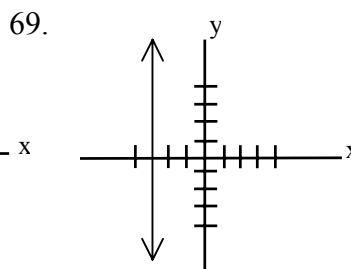
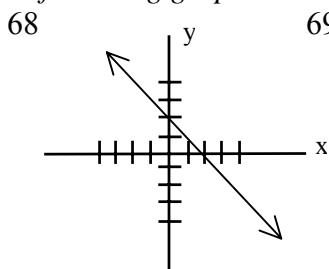
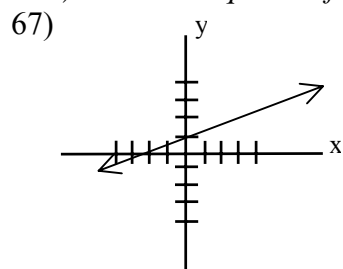
61) Find the equation of the vertical line that passes through the point (3, 2).

62) Find the equation of the horizontal line that passes through the point (1, -5).

VI) Find the intercepts.

- 63)
- 64)
- 65)
- 66)

VIII) Write the equation for the following graphs:



IX) Given the slope, sketch the following lines:

- 73) Sketch a line with a slope of 2.
- 74) Sketch a line with a slope of $\frac{1}{2}$.
- 75) Sketch a line with a slope of -2.
- 76) Sketch a line with a slope of $-\frac{1}{2}$.

X) Sketch the following graphs. Use parent functions and transformations when appropriate and/or plot points. Check your work with a calculator.

77) $y = 3x + 1$

78) $y = x^2$

79) $y = |x|$

80) $y = x^3$

81) $x = 3$

82) $y = -4$

83) $y = \ln x$

84) $y = \sqrt{x}$

85) $y = \frac{1}{x}$

86) $y = \frac{1}{x^2}$

87) $y = |x + 1|$

88) $y = x^2 + 2x - 3$

X) Sketch the following graphs continued

89) $y = x^3 + 1$

90) $y = (x + 1)^2$

91) $y = -x^2 + 1$

92) $y = (x + 1)^{\frac{1}{3}}$

93) $y = x^{\frac{2}{3}}$

94) $y = (x + 1)^{\frac{-1}{2}}$

95) $y = \log x$

96) $y = e^x$

97) $y = -\frac{1}{x}$

98) $x^2 + y^2 = 25$

99) $\frac{x^2}{9} + \frac{y^2}{4} = 1$

100) $\frac{x^2}{16} - \frac{y^2}{4} = 1$

101)
$$y = \begin{cases} -2 & \text{for } x < -1 \\ x & \text{for } -1 \leq x < 1 \\ 0 & \text{for } x \geq 1 \end{cases}$$

102)
$$y = \begin{cases} x + 1 & \text{for } x < 1 \\ x^2 & \text{for } x \geq 1 \end{cases}$$

XI) Rewrite the following functions without absolute values. In other words, rewrite as piecewise functions:

103) $y = |x|$

104) $y = |x - 1|$

105) $y = |x + 3| + 1$

106) $y = \frac{|x| - 2}{x - 2}$

XII) Find the domain and range of each function:

107) $y = x - 1$

108) $y = \frac{1}{x}$

109) $y = \frac{1}{x^2 + 1}$

110) $y = \frac{1}{\sqrt{x^2 + x + 1}}$

111) $y = \sqrt{x - 4}$

112) $y = |x - 1| + 2$

113) $y = \log x$

114) $y = 2^x$

115) $y = \sqrt{x^2 - 3x - 4}$

116)
$$y = \begin{cases} x + 3 & \text{for } x < 2 \\ x^2 & \text{for } x > 2 \end{cases}$$

XIII. Find the inverse of each function:

117) $f(x) = x + 3$

118) $f(x) = \sqrt{x}$

119) $f(x) = \frac{x}{x + 2}$

120)

XIV) Find the compositions of the function if $f(x) = x^3 + 1$, $g(x) = x^2 - 2$, and $h(x) = x + 3$

121) $f(2)$

122) $f(h(x))$

123) $f(g(2))$

124) $g(g(x))$

125) $g(f(x))$

126) $h(g(x))$

127) $h(x^2)$

128) $f(x+h)$

129) $h(p)$

130) $\frac{g(t+h) - g(t)}{h}$

XV) Solve the simultaneous equations:

131) $2x + 3y = 8$
 $x + 2y = 5$

132) $y = x^2 + 2x + 9$
 $7x + y = 19$

XVI) Intersection of curves:

132) Find the point of intersection between the lines $y = x + 1$ and $3y - x = 5$.

133) Find the points of intersection between the line $y = x + 7$ and the curve $x^2 + 2x = 5$.
Also sketch the area between the graphs.

XIII) What do the following mean if:

135) a graph is in the first quadrant

136) $f(2) = 5$ (another way to write it)

137) an expression is a function

138) a zero of a function is 4

139) a example of y directly proportional to x

140) an example where the coefficient of the third term is 5

141) a function only has one root

142) a function is a polynomial

143) two triangles are similar

XVIII) What are the following formulas?:

144) quadratic formula

145) pythagorean theorem

146) the hypotenuse of a 45-45-90 isosceles right triangle with a leg of length x

XVIII) What are the following formulas CONTINUED:

- 147) the hypotenuse of a 30-60-90 right triangle with shortest leg having a length of x
- 148) the volume of a sphere
- 149) the volume of a cylinder
- 150) the volume of a cone
- 151) the volume of a box with a square base
- 152) the surface area of a sphere
- 153) the surface area of a cylinder with no top
- 154) the area of a triangle
- 155) the area of a trapezoid
- 156)- the cross section through the center of a sphere
- 157) the volume of a prism that has an equilateral triangle with side of length x and height of length y

XIX) Solve by using similar triangles:

- 158) A six foot man is standing 10 feet away from a 20 foot lamppost. What is the length of his shadow?
- 159) Water is dripping out of a conical figure that has a diameter of 9 inches and a height of 12 inches. When the depth of the water is 8 inches, what is the radius of the water?

XX) Describe the symmetry of the following functions:

- 160) $y = x^2 + 1$
- 161) $y = x^3 + 1$
- 162) $y = |x|$
- 163) $y = x^3 + x$
- 164) $y = \sin x$
- 165) $y = \cos x$

XXI) Find the equations for the horizontal and vertical asymptotes of each function:

- 166) $y = \frac{1}{x-1}$
- 167) $y = \frac{x}{\sqrt{x^2-9}}$
- 168) $y = \frac{x^3}{x^3-1}$
- 169) $y = \frac{x}{\sqrt{x^2+1}}$

XXII) Write the following without sigma notation:

- 170) $\sum_{n=1}^4 2n + 1$
- 171) $\sum_{n=1}^4 2$

XXIII) Exponent rules: Which of the following are true? If false, explain why briefly.

172) $x^0 = 1$ 173) $x^{-2} = \frac{1}{x^2}$ 174) $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$

175) $x^5 \cdot x^3 = x^{15}$ 176) $x^5 \cdot x^5 = (xy)^5$ 177) $(x^3)^5 = x^8$

178) $x^{5-w} = \frac{x^5}{x^w}$ 179) $x^{t+s} = (x^t)^s$ 180) $\sqrt{\frac{9}{4}} = \frac{3}{2}$

181) $\left(4x^{\frac{1}{2}}\right) = 2x$ 182) $\sqrt{\frac{1}{x}} = x^{\frac{-1}{2}}$ 183) $\sqrt{x^2} = x$

184) $\sqrt{x^2 - 1} = x - 1$ 185) $x^{\frac{4}{3}} = \sqrt[4]{x^3}$ 186) $\left(x^{\frac{1}{2}} + y^{\frac{1}{2}}\right)^2 = x + y$

187) $x^{\frac{-2}{3}} = \frac{1}{\sqrt[3]{x^2}}$ 188) $e^{\ln x^2} = x^2$ 189) $\ln e^3 = 3$

190) $e^{2 \ln 2 - \ln 4} = \frac{4}{5}$ 191) $\ln x^2 = (\ln x)^2$

XXIV) Do these on a graphing calculator:

192) Graph $y = .1x^3 + 2x^2 - x - 3$ with domain $[-4, 4]$ and range $[-4, 4]$

193) Find the roots of the equation above.

194) Find the point of intersection for the graphs $y = x^3 + x - 3$ and $y = 2x + 4$.

195) Find the maximum value for the graph $f(x) = -x^4 + x - 4$

196) For $f(x) = -x^4 + x - 4$, on what intervals for x is f increasing?

XXV) Write the trigonometric identities for the following?

197) $\sec x$ 198) $\csc x$ 199) $\tan x =$

200) $\cot x$ 201) $\cos^2 x - 1 =$ 202) $\sec^2 x - 1 =$

203) $\cot^2 x + 1$ 204) 205)

206) (double angle) $\sin 2x$ 207) (double angle) $\cos 2x$ 208) $\sin(A+B)$

209) $\cos(a+b)$

XXVI) Find the period of the following functions:

210) $y = 4\sin(2x) + 1$

211) $y = -3\tan(\pi x)$

212) $y = \cos^2 x$

213) $y = -3\sec(6x) + 2$

XXVII) Evaluate the following expressions:

214) $\sin\left(\frac{\pi}{6}\right)$

215) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

216) $\tan 210^\circ$

217) $\cos(0)$

218) $\cos 45^\circ$

219) $\csc\left(\frac{-5\pi}{6}\right)$

220) $\sec 180^\circ$

221) $\cot\left(\frac{-2\pi}{2}\right)$

222) $\sin^{-1}\left(\frac{1}{2}\right)$

223) $\tan\left(\frac{\pi}{6}\right)$

224) $\sin^2\left(\frac{5\pi}{6}\right)$

225) $\cot\left(\frac{2\pi}{3}\right)$

226) $\sin\left(\frac{\pi}{2}\right)$

227) $\cot^{-1}(-1)$

228) $\sec\left(\frac{3}{4}\right)$

229) $\tan^{-1}(-1)$

230) $\sec 180^\circ$

231) $\sec^2\left(\frac{-11\pi}{6}\right)$

232) $\sin(\csc^{-1}(-2))$

233) $\sec^{-1}(\sec 30^\circ)$

XXVIII) Sketch one period of the following trigonometric graphs:

234) $y = \sin x$

235) $y = \cos x$

236) $y = \tan x$

237) $y = \sec x$

238) $y = \csc x$

239) $y = \cot x$

240) $y = \sin 2x$

241) $y = 3\cos 4x - 1$

242) $y = \cos^2 x$

243) $y = |\sin x|$

244) $y = \cos \pi x$

245) $y = \cos(-\pi x)$

XXIX) Solve the following trigonometric equations for the given domain:

246) $\sin x = \cos x$ on $[0, 2\pi]$

247) $\sin 2x = \frac{1}{2}$ on $[0, 2\pi]$

248) $\sin 3x = \frac{\sqrt{3}}{2}$ for all x

249) $\cos 4x = -\frac{1}{2}$ on $[0, 2\pi]$

250) $\sin^2 x + \sin x = 0$ on $[0, 2\pi]$

251) $\tan^2 x = \sqrt{3}$ on $[0, 2\pi]$

252) $\sec x \csc x - 2\csc x = 0$ on $[0, 2\pi]$

253) $2\cos^2 x - \cos x = 1$ on $[0, 2\pi]$

254) $\cos^2 \theta + \sin \theta = 1$ on $[0, 2\pi]$

254) $\cos \frac{\theta}{2} - \cos \theta = 1$ on $[0, 2\pi]$