# Integrating with Natural Log Functions

## Part II

It pays to remember our natural log rules:

$$\int \frac{dx}{x\ln\left(x^2\right)}$$

$$= \int \frac{dx}{2x\ln x}$$

$$u = \ln x$$
$$du = \frac{1}{x}\,dx$$

$$= \frac{1}{2}\int \frac{1}{x\ln x}\,dx$$

$$= \frac{1}{2}\int \left(\frac{1}{\ln x}\right)\left(\frac{1}{x}\right)dx$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|\ln x| + C$$

And now we can find some trig integrals that we were unable to do until now:

$\int \tan x\,dx$

$$= \int \frac{\sin x}{\cos x}\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$$= -\int \frac{1}{u}\,du$$

$$= -\ln|u| + C$$

$$= -\ln|\cos x| + C$$

FANCY SKI PANTS ANSWER

$$= \ln|\sec x| + C$$

You try: $\int \cot x\,dx$

$$= \int \frac{\cos x}{\sin x}\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$$= \int \frac{1}{u}\,du$$

$$= \ln|u| + C$$

$$= \ln|\sin x| + C$$

$\int \tan(7x)\,dx$

$u = \cos(7x)$

$du = -7\sin(7x)\,dx$

$-\frac{1}{7}du = \sin(7x)\,dx$

$= \int \frac{\sin(7x)}{\cos(7x)}\,dx$

$= -\frac{1}{7}\int \frac{1}{u}\,du$

$= -\frac{1}{7}\ln|\cos(7x)| + C$

$\int \frac{\sec^2 x}{\tan x + 5}\,dx$

$u = \tan x + 5$

$du = \sec^2 x\,dx$

$= \int \frac{1}{u}\,du$

$= \ln|\tan x + 5| + C$

A really weird integral!

$\int \sec x \, dx$

We can multiply by a clever form of one

$$=\int \sec x \frac{\sec x+\tan x}{\sec x+\tan x}\, dx$$

$$=\int \frac{\sec^2 x+\sec x\tan x}{\sec x+\tan x}\, dx$$

Let $u=\sec x+\tan x$

$du=[\sec x\tan x+\sec^2 x]\,dx$

$$=\int \frac{1}{u}\, du$$

$$=\ln|u|+C$$

$$=\ln|\sec x+\tan x|+C$$

Definite Integrals

$$\int_1^2 \frac{1-\cos\theta}{\theta-\sin\theta}\, d\theta$$

Let $u = \theta - \sin\theta$

$du = (1-\cos\theta)\, d\theta$

$u(1) = 1 - \sin 1$

$u(2) = 2 - \sin 2$

$$= \int_{1-\sin 1}^{2-\sin 2} \frac{1}{u}\, du$$

$$= \ln|u| \Big|_{1-\sin 1}^{2-\sin 2}$$

$$= \ln|2-\sin 2| - \ln|1-\sin 1|$$

$$= \ln\left|\frac{2-\sin 2}{1-\sin 1}\right|$$

You try:

$$\int_1^e \frac{(1+\ln x)^2}{x}\, dx$$

$u = 1 + \ln x$

$du = \frac{1}{x}\, dx$

$u(1) = 1$

$u(e) = 2$

$$= \int_1^2 u^2\, du$$

$$= \frac{1}{3}u^3 \Big|_1^2$$

$$= \frac{1}{3}(8-1)$$

$$= \frac{7}{3}$$

Let's not forget the Second FTC!

Let $F(x) = \int_1^x \frac{1}{t}\, dt$ Find $F'(x)$

$$F'(x) = \frac{d}{dx}\int^{x} \frac{1}{t}\, dt$$

STRAWBERRY

$$F'(x) = \frac{1}{x}$$

Let $F(x) = \int_1^{3x} \frac{1}{t}\, dt$ Find $F'(x)$

$$F'(x) = \frac{d}{dx}\int_1^{3x} \frac{1}{t}\, dt$$

RASPBERRY

$$F'(x) = \frac{1}{3x}(3)$$

$$F'(x) = \frac{1}{x}$$

Let $F(x) = \int_{\frac{\pi}{4}}^{x} \cot t \, dt$ Find $F'(x)$

$$F'(x) = \frac{d}{dx} \int_{\frac{\pi}{4}}^{x} \cot t \, dt$$

$$F'(x) = \cot x$$

Please remember to bring your calculator for block day.

Homework: page 338 #33, 35 and page 339 #47, 51, 53

use Algebra