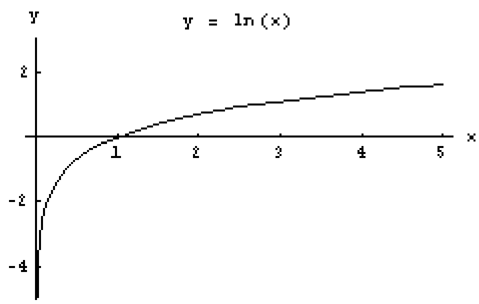
# The Natural Log Function $f(x) = \ln x$

Can't remember the rules of $f(x) = \ln x$? Go to our webpage or to http://www.rapidtables.com/math/algebra/Ln.htm




$y = \ln x$
$(1,0), (e,1)$

$$\lim_{x \to 0^+} \ln x = -\infty$$
$$\lim_{x \to \infty} \ln x = \infty$$

$\dfrac{d}{dx} \ln x = \dfrac{1}{x}$ which means that $\int \dfrac{1}{x}\,dx = \ln x + C$

Note: We might have domain issues that we will consider in the next class.

$$\frac{d}{dx}\int_1^x \ln t\,dt = \ln x$$
$$\frac{d}{dx}\int_1^x \frac{1}{t}\,dt = \frac{1}{x}$$

2nd FTC

Now that we know that $\frac{d}{dx}\ln x = \frac{1}{x}$ on $(0,\infty)$ we can analyze!!!

Does $f(x) = \ln x$ have any critical values on its domain?

NO

On $(0,\infty)$, $f'(x) = \frac{1}{x}$

$f'(x) > 0$, $f(x) = \ln x$ is ALWAYS increasing

If $f(x) = \ln x$, then $f''(x) = -\frac{1}{x^2}$ and on $(0,\infty)$ $f''(x) < 0$, therefore $f(x) = \ln x$ is ALWAYS CONCAVE frown

What is the average rate of change of $f(x) = \ln x$ on $[1, e]$?

$$\text{ARof}\,\Delta \text{ on } [1,e] = \frac{f(e) - f(1)}{e - 1}$$

$$= \frac{1}{e-1}$$

What is the average value of $f(x) = \ln x$ on $[1, e]$?

$$AV = \frac{1}{e-1}\int_1^e \ln x \, dx$$

$$= \frac{1}{e-1}$$

The Product Rule and the Quotient Rule along with the Chain Rule and u-sub still apply! Caution: you may want to simplify if you can before differentiating or integrating.

$\frac{d}{dx}\ln x^2$ [Can you simplify first?]

$$= \frac{d}{dx}(2\ln x)$$

$$= \frac{2}{x}$$

$$\frac{d}{dx}\left(\underline{x^2}\ \underline{\underline{\ln x}}\right)$$

$$= 2x\ln x + \left(\frac{1}{x}\right)(x^2)$$

$$= 2x\ln x + x$$

$\frac{d}{dx}\ln(2x)$ [Can you simplify?]

$$= \frac{d}{dx}[\ln 2 + \ln x]$$

$$= \frac{1}{x}$$

$$\frac{d}{dx}\ln(5x)$$

$$= \frac{d}{dx}[\ln 5 + \ln x]$$

$$= \frac{1}{x}$$

$$\frac{d}{dx}\ln\left(x^2+1\right)$$

$$u = x^2+1$$

$$\frac{du}{dx} = 2x$$

$$= \frac{d}{dx}\ln u$$

$$= \frac{1}{u}\frac{du}{dx}$$

$$= \frac{2x}{x^2+1}$$

$$\frac{d}{dx}\ln\left(\frac{2}{x^2}\right)$$

[Can you simplify?]

$$= \frac{d}{dx}[\ln 2 - \ln x^2]$$

$$= \frac{d}{dx}[\ln 2 - 2\ln x]$$

$$= \frac{-2}{x}$$

Let $f(x) = \int_2^{\ln 2x}(t+1)\,dt$ and find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left[\int_2^{\ln 2x}(t+1)\,dt\right]$$ RASPBERRY

$$f'(x) = (\ln 2x + 1)\left(\frac{1}{x}\right)$$

Finding equations of tangent lines – we still need the same two items: a point and a slope

Find the equation of the line tangent to the graph of $f(x) = 3x^2 - \ln x$ at $(1,3)$

slope: $f'(x) = 6x - \frac{1}{x}$

$f'(1) = 5$

$y - 3 = 5(x-1)$

## Implicit Differentiation

$$\frac{d}{dx}\ln y = \frac{1}{y}\frac{dy}{dx}$$

Consider the curve: $x^2 - 3\ln y + y^2 = 0$ Find $\frac{dy}{dx}$

$$\frac{d}{dx}x^2 - \frac{d}{dx}3\ln y + \frac{d}{dx}y^2 = \frac{d}{dx}0$$

$$2x - 3\left(\frac{1}{y}\right)\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(2y - \frac{3}{y}\right) = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y - \frac{3}{y}}$$

Consider the curve $\ln(xy) + 5x = 30$ Find $\frac{dy}{dx}$

$$\frac{d}{dx}\ln(xy) + \frac{d}{dx}5x = \frac{d}{dx}(30)$$

$$\frac{d}{dx}\ln x + \frac{d}{dx}\ln y + \frac{d}{dx}5x = \frac{d}{dx}(30)$$

$$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} + 5 = 0$$

$$\frac{1}{x} + 5 = -\frac{1}{y}\frac{dy}{dx}$$

You try:

$$-y\left(\frac{1}{x}+5\right) = \frac{dy}{dx}$$

$$\frac{d}{dx}\ln x^4$$

$$= \frac{d}{dx}(4\ln x)$$

$$= \frac{4}{x}$$

$$\frac{d}{dx}\left(x^4 \ln x\right)$$

$$= 4x^3 \ln x + (\tfrac{1}{x})$$

________ ____________

____________ $(x^4)$

$$\frac{d}{dx}\ln(\cos x)$$

$$= \frac{d}{dx}\ln u \qquad u = \cos x$$

$$= \frac{1}{u}\frac{du}{dx} \qquad \frac{du}{dx} = -\sin x$$

$$= \frac{1}{\cos x}(-\sin x)$$

$$= -\tan x$$

$$\frac{d}{dx}\left[\frac{\ln x}{x^4}\right]$$

$$= \frac{(x^4)\left(\frac{1}{x}\right) - (\ln x)(4x^3)}{x^8}$$

$$\frac{d}{dx}\sqrt{\ln x}$$

$$= \frac{d}{dx}\sqrt{u} \qquad u = \ln x$$

$$= \frac{1}{2\sqrt{u}}\frac{du}{dx} \qquad \frac{du}{dx} = \frac{1}{x}$$

$$= \frac{1}{2x\sqrt{\ln x}}$$

$$\frac{d}{dx}[\ln(2y)]$$

$$= \frac{d}{dx}\left[\ln 2 + \ln y\right]$$

$$= \frac{1}{y}\frac{dy}{dx}$$

Homework: page 330 #46, 48, 54, 56, 64, 70, 80