Today's Announcements:

Free Response Test Next Block Day

Physics Extravaganza
(In the style of: Gilbert & Sullivan - Modern Major General)

Position is the place you are
at any given time you see.
The instantaneous rate of change
of that is the velocity.

$s'(t)$
$= v(t)$

Which is direction and the speed
two parts of information.
Its instantaneous rate of change
is called acceleration.

$v'(t)$
$= a(t)$

The total distance traveled
is by no means an atrocity,
the integral of absolute value of the velocity!
Another point of interest know
the integral of force is work.

TDT on $[a, b]$

$$= \int_a^b |v(t)|\,dt$$

Acceleration's rate of change is
surge or lurch or jolt, or jerk!

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$a'(t) =$
JERK

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Acceleration's rate of change is
surge or lurch or jolt, or jolt or jerk!

Displacement is how far you are
from your initial starting spot.
Remembering the average value
of a function is a lot:

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

One over B minus A times the value of the integral
from A to B of f of x dx if on an interval.

SING

One over B minus A times the value of the integral
from A to B of f of x dx if on an interval!

4. 2008 AB2 [calculator]

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

$$\frac{1}{4-0}\int_0^4 L(t)\,dt$$

$$\int_0^4 L(t)\,dt \approx \text{TRAP}$$

$$\text{TRAP} = \frac{L(0)+L(1)}{2}(1) + \frac{L(1)+L(3)}{2}(2) + \frac{L(3)+L(4)}{2}(1)$$

$$= 621 \text{ (people)(hour)}$$

$$\text{Av value} = \frac{1}{4}(621) = 155.25 \text{ people}$$

Average Rate versus Average Value

[Why you need to be a careful reader]

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

$$\text{AV ACC on } [0, 80] = \frac{V(80) - V(0)}{80 - 0} \frac{\frac{ft}{sec}}{sec}$$

$$= \frac{44}{80} \frac{ft}{sec^2}$$

(b) Find the average of the velocity for $0 \le t \le 80$ seconds using a Right Riemann sum.

$$\text{AV vel on } [0,80] = \frac{1}{80-0} \int_0^{80} v(t)\,dt$$

$$\int_0^{80} V(t)\,dt \approx RRAM$$

$$RRAM = 10\left[v(10) + v(20) + V(30) + V(40) + v(50) + v(60) + v(70) + v(80)\right]$$

$$= 2800 \text{ ft}$$

$$\text{AV vel} = \frac{1}{80}(2800)$$

$$= 35 \frac{ft}{sec}$$

Two particles move along the x-axis. For $0 \le t \le 6$, the position of particle P at time t is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \le t \le 3$

distance $= |p(t) - r(t)|$

$$\frac{1}{3-1}\int_1^3 |p(t) - r(t)|\,dt$$

For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

$$\frac{1}{6-0}\int_0^6 v(t)\,dt$$

≈ 1.949

Find the position of the particle at time $t = 3$

[It is not an "average" question, but since we have this nice problem, then …]

$$x(3) = x(0) + \int_0^3 v(t)\,dt$$

Another motion problem:

A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t) = \ln\left(t^2 - 3t + 3\right)$. The particle is at position $x = 8$ at time $t = 0$.

(d) Find the average speed of the particle over the interval $0 \le t \le 2$.

$$\text{AV speed over } [0,2] = \frac{1}{2-0}\int_0^2 |v(t)|\,dt$$

$$\approx .3705$$

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

$$\frac{1}{15-10}\int_{10}^{15} F(t)\,dt$$

$$\approx 81.899 \ \frac{\text{cars}}{\text{min}}$$

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

$$\text{AV R of }\Delta = \frac{F(15)-F(10)}{15-10} \quad \frac{\frac{\text{cars}}{\text{min}}}{\text{min}}$$

$$\approx 1.517 \text{ or } 1.518 \ \frac{\text{cars}}{\text{min}^2}$$

A 1200 ml cauldron [big pot] of Polyjuice Potion is filled to capacity. [You can find the recipe at: http://harrypotter.wikia.com/wiki/Polyjuice_Potion]

At time t = 0, the potion magically begins to drain out of the cauldron at a rate modeled by $r(t)$,measured in milliliters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{50t}{t+5} & \text{for } 0 \le t \le 5 \\ 70e^{-0.2t} & \text{for } t > 5 \end{cases}$$

this is a rate!

(b) Find the average rate at which the Polyjuice potion is draining from the cauldron from the tank between time $t=0$ and time $t=10$

$$\frac{1}{10-0}\int_0^{10} r(t)\,dt$$

$$\frac{1}{10}\left[\int_0^5 \frac{50t}{t+5}\,dt + \int_5^{10} 70e^{-0.2t}\,dt\right]$$

$$\approx 15.81036601 \ \frac{ml}{hr}$$