The exponential function e^x

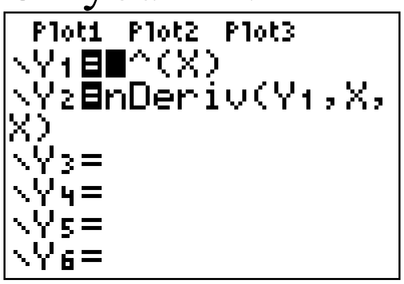
If $f(x) = \ln x$, then $f^{-1}(x) = e^x$

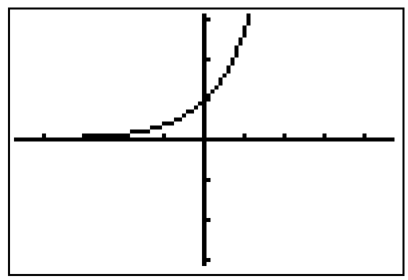
For $y = e^x$: Domain: $(-\infty, \infty)$ and Range : $(0, \infty)$

$$\lim_{x\to\infty} e^x = \infty$$

$$\lim_{x\to-\infty} e^x = 0 \qquad\qquad ♪\ e^x > 0$$

On your TI:





$$\frac{d}{dx}e^x = e^x$$ It's a miracle!

Now that we know that $\frac{d}{dx}e^x = e^x$ we can analyze the graph of $f(x) = e^x$!

$f(x) = e^x$

$f'(x) = e^x$

Are there any critical values? NO

Conclusion:

Since $f'(x) = e^x > 0$, then $f(x) = e^x$ is always increasing

$f''(x) = e^x$

Are there any possible points of inflection? NO

Conclusion: $f''(x) = e^x > 0$ then $f(x) = e^x$ is ALWAYS CONCAVE up

All of the rules of derivatives and integration still hold.

Here is a four star rule: ✵✵✵✵ $\frac{d}{dx} e^u = e^u \frac{du}{dx}$

Solving equations with e^x or $\ln x$

1. $e^{\ln 2x} = 12$ Simplify!

   $2x = 12$ Now solve.

   $x = 6$ Feel free to check

2. $4e^x = 12$ Isolate

   $e^x = 3$ Take ln of both sides

   $\ln e^x = \ln 3$ Simplify and solve

   $x = \ln 3$

3. $\ln x = 2$ Exponentiate both sides

   $e^{\ln x} = e^2$ Simplify and solve

   $x = e^2$

4. $\ln(x-3)=2$ Exponentiate both sides

$$e^{\ln(x-3)} = e^2$$ Simplify and solve

$$x-3 = e^2$$

$$x = e^2+3$$

5. $\ln\sqrt{x+2} = 1$ Exponentiate both sides

$$e^{\ln\sqrt{x+2}} = e^1$$ Simplify

$$\sqrt{x+2} = e$$ Solve

$$x+2 = e^2$$

$$x = e^2-2$$

Now let's consider some Calculus using our new friend, e^x !

$$\frac{d}{dx}\left(\underline{x^2}\,\underline{\underline{e^x}}\right)$$

$$= 2xe^x + x^2e^x$$

What rule?

PRODUCT

$$\frac{d}{dx}\left(\frac{x^2}{e^x}\right)$$

What rule? Quotient

$$= \frac{e^x(2x) - x^2 e^x}{e^{2x}}$$

$$\frac{d}{dx}\left(e^{5x}\right)$$

What rule? CHAIN

Let's use our new four star rule: ✵✵✵✵ $\frac{d}{dx}e^u = e^u \frac{du}{dx}$

$$= \frac{d}{dx} e^u \qquad u = 5x$$

$$= e^u \frac{du}{dx} \qquad \frac{du}{dx} = 5$$

$$= 5e^{5x}$$

$$\frac{d}{dx}\int_2^x e^t \, dt$$

2^{nd} FTC again!

STRAWBERRY

$$= e^x$$

$$\frac{d}{dx}\int_2^{x^2} e^t \, dt$$

RASPBERRY

$$= 2xe^{x^2}$$

$$\frac{d}{dx}e^{-x} = \frac{d}{dx}e^{u}$$
$$= e^{u}\frac{du}{dx}$$
$$= -e^{-x}$$

$u = -x$

$\frac{du}{dx} = -1$

Another ✵✵✵✵ rule! $\int e^x dx = e^x + C$

Find the average rate of change of $f(x) = e^x$ on [0, 1]

Av R of Δ on [0, 1]

$$= \frac{f(1)-f(0)}{1-0}$$
$$= \frac{e^1 - e^0}{1}$$
$$= e - 1$$

Now find the average value of $f(x) = e^x$ on [0, 1]

Av Value on [0, 1]

$$= \frac{1}{1-0}\int_0^1 e^x dx$$
$$= e^x\Big|_0^1$$
$$= e - 1$$

Try not to get fooled by functions involving $\ln x$ and e^x

$\frac{d}{dx}\left(\ln\left(e^{x^2}\right)\right)$ Take a deep breath and simplify first.

$$= \frac{d}{dx}x^2$$
$$= 2x$$

And now a little implicit differentiation to round out our e^x knowledge:

$$\frac{d}{dx}e^y = e^y\frac{dy}{dx}$$

Find the equation of the line tangent to the curve $xe^y + ye^x = 1$ at the point (0, 1)

$$\frac{d}{dx}\underline{x}\,\underline{\underline{e^y}} + \frac{d}{dx}\underline{y}\,\underline{\underline{e^x}} = \frac{d}{dx}(1)$$

$$e^y + xe^y\frac{dy}{dx} + e^x\frac{dy}{dx} + ye^x = 0$$

$$\frac{dy}{dx}[xe^y + e^x] = -e^y - ye^x$$

$$\frac{dy}{dx} = \frac{-e^y - ye^x}{xe^y + e^x}$$

$$\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{-e-1}{0+1}$$

EQUATION OF
Tan line at (0,1)

$$y - 1 = (-e-1)(x-0)$$

$$e^{\ln x} = x \qquad \ln e^{x} = x$$

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Homework: pages 356, 357, 358 # 33, 35, 37, 45, 47, 49, 53, 57, 69