Some warm-up problems

If $h(x) = 7x + \int_3^{\sin x} \sqrt{t^2+2}\, dt$, *then what is the value of* $h'(7)$

$$h'(x) = \frac{d}{dx}\left[7x + \int_3^{\sin x} \sqrt{t^2+2}\, dt\right]$$

$$= 7 + \cos x \sqrt{\sin^2 x + 2}$$

$$h'(7) = 7 + \cos 7 \sqrt{\sin^2 7 + 2}$$

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$\int h'(\pi x)\, dx =$

$u = \pi x$

$du = \pi\, dx$

$\frac{1}{\pi} du = dx$

$$= \frac{1}{\pi} \int h'(u)\, du$$

$$= \frac{1}{\pi} h(u) + C$$

$$= \frac{1}{\pi} h(\pi x) + C$$

1. Non-calculator

Water is pumped into a tank at a constant rate of 12 gallons per minute. Water leaks out of the tank at a rate of $2t\sqrt{t^2+1}$, for $0 \le t \le 8$ minutes. At time $t=0$ the tank contains 120 gallons of water.

(a) How many gallons of water leak out of the tank from $t=0$ to $t=8$ minutes?

$$\int_0^8 2t\sqrt{t^2+1}\,dt$$

$u = t^2+1$
$du = 2t\,dt$
$u(0) = 1$
$u(8) = 65$

$$= \int_1^{65} \sqrt{u}\,du$$

$$= \frac{2}{3}u^{\frac{3}{2}}\Big|_1^{65}$$

$$= \frac{2}{3}\left(65^{3/2} - 1\right)$$

(b) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t

$$a(t) = 120 - \int_0^t 2x\sqrt{x^2+1}\,dx + \int_0^t 12\,dx$$

or $$120 + 12t - \int_0^t 2x\sqrt{x^2+1}\,dx$$

(c) How many gallons of water are in the tank at time $t=8$

$$a(8) = 120 - \int_0^8 2x\sqrt{x^2+1}\,dx + \int_0^8 12\,dx$$

$$= 120 - \frac{2}{3}\left(65^{\frac{3}{2}}-1\right) + 96$$

2. Calculator [Tea is made from water!]

t(minutes)	0	3	7	9	10
$H(t)$ °C	90	80	65	61	60

Harry Potter is having tea with his friends. As the pot of tea cools, the temperature is modeled by a differentiable function H for $0 \le t \le 10$ where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Selected values of $H(t)$ are given in the table above.

(a) Use the data from the table to approximate the rate at which the temperature of the tea is changing at time $t=7.5$. Show the computations that lead to your answer.

$$H'(7.5) \approx \frac{H(9)-H(7)}{9-7}$$

$$= -2 \frac{°C}{\text{min}}$$

(b) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

$$\int_0^{10} H'(t)\,dt$$
$$= H(t)\Big|_0^{10}$$
$$= H(10) - H(0)$$
$$= -30°C$$

this is the net change in Temp [°C] During $0 \le t \le 10$ min

Tea cooled by 30°C

(c) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$. Use a trapezoidal sum with the four subintervals indicated by the data to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$

SKIP UNTIL NEXT WEEK

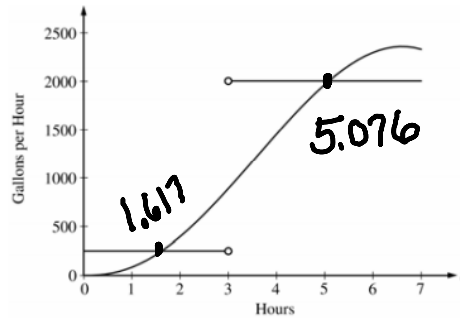
3. Calculator

The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is $f(t) = 100t^2 \sin(\sqrt{t})$ gallons per hour for $0 \le t \le 7$.

(ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases} \text{ gallons per hour.}$$



The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

(a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

(c) For $0 \le t \le 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

$$\int_0^7 f(t)\,dt \approx 8264 \text{ gallons}$$

the amount of water is decreasing on the intervals $0 \le t \le 1.617$ and $3 \le t \le 5.076$ because $f(t) < g(t)$ on $0 \le t < 1.617$ and $3 < t < 5.076$

Since $f(t) - g(t)$ changes from positive to negative values at $t = 3$ so our candidates are

$t = 0, t = 3, t = 7$

at $t=0$ 5000 gallons

at $t=3$ $5000-250(3)+\int_0^3 f(t)\,dt$

≈ 5127 gallons

at $t-7$ $5000-2000(4)-250(3)+\int_0^7 f(t)\,dt$

≈ 4514 gallons

the amt of H_2O is greatest at $t=3$

there is 5127 gallons at $t=3$

4. Calculator

A water tank at Camp Newton holds 1200 gallons of water at time $t = 0$. During the time interval $0 \le t \le 18$ hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t}\sin^2\left(\frac{t}{6}\right) \text{ gallons per hour.}$$

During the same time interval, water is removed from the tank at the rate

$$R(t) = 275\sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

(a) Is the amount of water in the tank increasing at time $t = 15$? Why or why not?

(b) To the nearest whole number, how many gallons of water are in the tank at time $t = 18$?

(c) At what time t, for $0 \le t \le 18$, is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.

(d) For $t > 18$, no water is pumped into the tank, but water continues to be removed at the rate $R(t)$ until the tank becomes empty. Let k be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

(a) since $w(15)-R(15)\approx -121.09 < 0$, then,

NO, the amount of H_2O is NOT increasing

(b) $1200-\int_0^{18} R(x)\,dt+\int_0^{18} w(x)\,dx$

≈ 1310 gallons

(c) $W(t) = R(t)$ at

$t \approx 6.495$ rel min

$t \approx 12.975$ rel max

CANDIDATES: $t=0$, $t=18$, $t \approx 6.495$

$t=0$ 1200 gallons

$t=18$ 1310 gallons

$t=6.495 \approx 525$ gallons

the ab min occurs at $t \approx 6.495$ with only 525 gallons

(d) $\int_{18}^{k} R(t)\,dt \approx 1310$