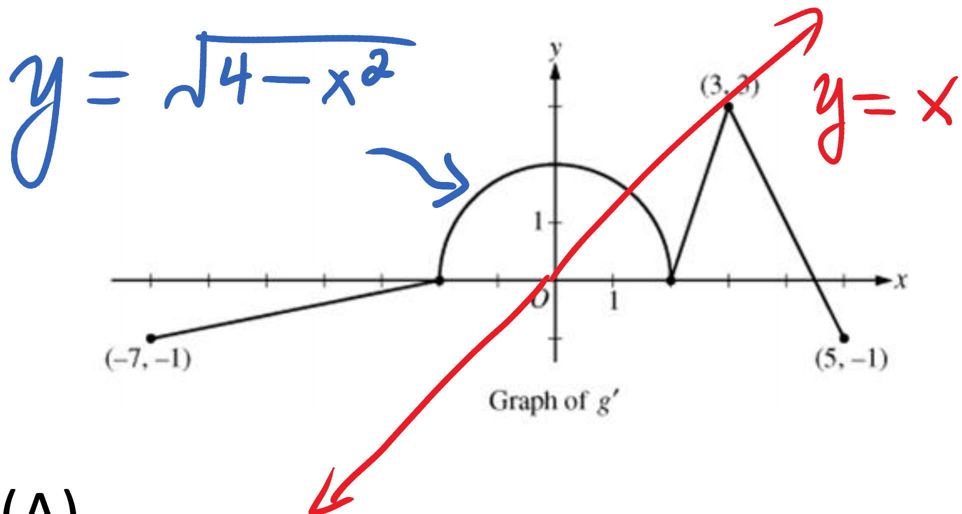
Solutions

2010 AP® CALCULUS AB FREE-RESPONSE QUESTIONS



Graph of g'

(A)

Think "initial value + accumulated rate of change"

$$g(3) = g(0) + \int_0^3 g'(x)\,dx$$

$$= 5 + \frac{4\pi}{4} + \frac{3}{2}$$

$$= \frac{13}{2} + \pi$$

$$g(-2) = g(0) + \int_0^{-2} g'(x)\,dx$$

$$= g(0) - \int_{-2}^{0} g'(x)\,dx$$

$$= 5 - \pi$$

(B) *We are given the graph of g', so use it.*

The graph of g' changes from increasing to decreasing at x= 0 and x= 3. The graph of g' changes from decreasing to increasing at x=2. Hence g has points of inflection at x=0, x=2 and x=3

(C) A return to the glory of Chapter 3!

$$h(x) = g(x) - 0.5x^2$$

$$h'(x) = g'(x) - x$$

$h'(x) = 0$ if $g'(x) = x$ so draw the line $y = x$ onto your graph and look for intersection(s)

$g'(x) = x$ at $x = 3$ and $x = \sqrt{2}$

Now look at your graph

$(-7, \sqrt{2})$ $(\sqrt{2}, 3)$ $(3,5)$

$h' > 0$ $h' < 0$ $h' < 0$

At $x = \sqrt{2}$ h' changes from positive to negative values, hence h has a relative maximum at $x = \sqrt{2}$.

At x=3 h' does not change from either negative to positive or positive to negative, hence h has neither a rel. min. or relative maximum at x = 3.

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

$$E'(6) \approx \frac{E(7) - E(5)}{7 - 5}$$

$$= 4 \ \frac{\text{hundred entries}}{\text{hour}}$$

(B) We already did this part so consult your previous notes.

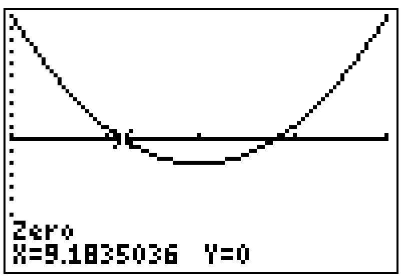
(C)

$$23-\int_{8}^{12} P(t)\,dt = 7$$ hundred entries

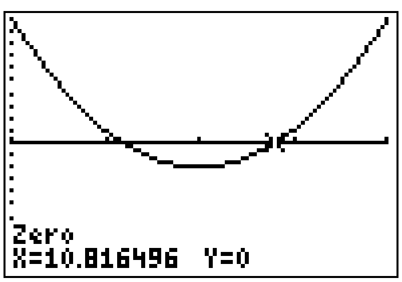
↑ VALUE FROM TABLE

(D) Another stab at Chapter 3 topics.

Use your TI!



P'(t)=0



Our candidates:

$t=8,\ t\approx 9.183503,\ t\approx 10.816496,\ t=12$

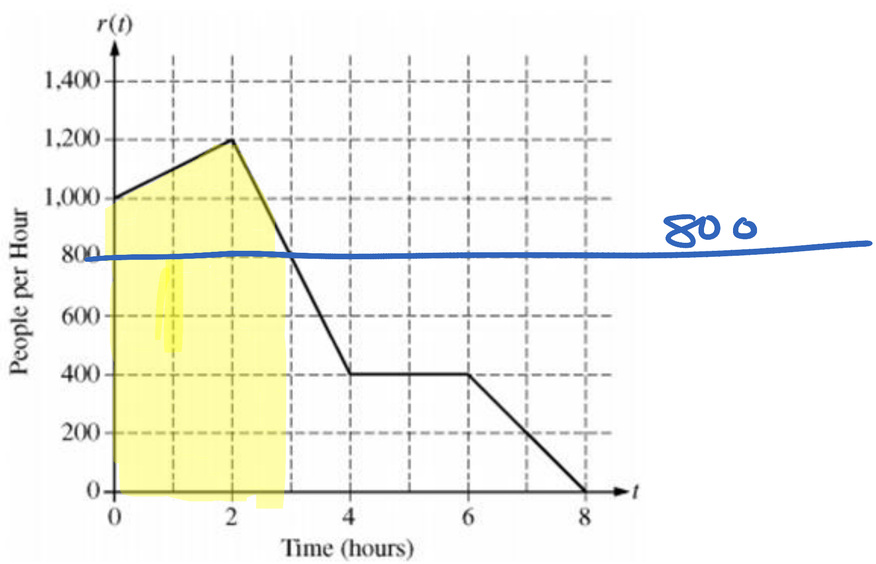
P(8)=0

$P(9.183503) \approx 5.088662$

$P(10.816497) \approx 2.911338$

$P(12) = 8$

Hence, the entries were processed most quickly at t=12 [or midnight]



You can think of this as a variation of the Sandy Point Beach problem. You are given an initial value, a rate at which people are getting into the line, and a rate at which people are getting out of the line [they are getting onto the ride]

If you let $P(t)$ be the number of people in line, then $P(t) = 700 + \int_0^t r(x)dx - \int_0^t 800dx$

(A) $\int_0^3 r(t)dt = \frac{1000+1200}{2}(2) + \frac{1200+800}{2}(1)$

$= 3200 \ people$

(B) The number of people waiting in line is increasing because for 2<t<3, r(t)>800 [people move onto the ride at a rate of 800 people/hour] See the blue line on the graph.

(C) r(t)=800 at t=3

$0 \le t < 3$ $3 < t \le 8$

r(t)>800 r(t)<800

Hey! A relative maximum at t=3.

At time t=3 there are

700+$\int_0^3 r(x)\,dx - \int_0^3 800\,dx$=1500 people

Initial amount – people already on ride

+number of people waiting in line

0=700+$\int_0^t r(x)\,dx - \int_0^t 800\,dx$

x	2	3	5	8	13
$f(x)$	1	4	–2	3	6

(A)

$$f'(4) \approx \frac{f(5)-f(3)}{5-3}$$

$$= -3$$

(B)

$$\int_2^{13} [3-5f'(x)]\,dx$$

$$= \int_2^{13} 3\,dx - 5\int_2^{13} f'(x)\,dx$$

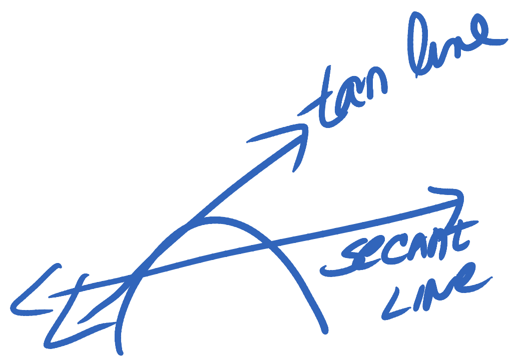
$$= 3x\Big|_2^{13} - 5[f(13)-f(2)]$$

$$= 8$$

(C) $\int_2^{13} f(x)\,dx \approx LRAM$

$LRAM = 1f(2) + 2f(3) + 3f(5) + 5f(8)$
$= 18$

(D) Equation of tangent line



$y + 2 = 3(x - 5)$

f''(x)<0 [f is concave down] so the tangent line lies above the graph of f(x) so

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$f(7) \le -2 + 3(2) = 4$

Secant line equation

$y + 2 = \frac{5}{3}(x - 5)$

Secant line lies below the curve of f(x) because f''(x)<0 [concave down]

$$f(7) \geq -2 + \frac{5}{3}(2) = \frac{4}{3}$$