17 January 2012

Today's Announcements:

Gryffindor now in first place!

Quiz Results:

2^{nd}	**52% A's**
4^{th}	**58% A's**
5^{th}	**61% A's**
7^{th}	**71% A's**

Next Week:

Free Response Test

Question that was missed the most on the quiz:
If f is a linear function and $0 < a < b$, then

$$\int_b^a f''(x)\,dx = 0$$

Average Value of a Function

How do you usually find a mean [average]?
Do not confuse with average rate of change [which is a slope].

Average value of a function is a value of $f(c)$.

If f is integrable [can be integrated] on $[a,b]$ then the average value of f is

$$\text{Average value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

You need to memorize this!!!!

Let's try one:

Let $f(x)=1-x^2$ on $\underset{[a,\,b]}{[-2,2]}$. Find the average value.

Our formula is $\frac{1}{b-a}\int_a^b f(x)\,dx$

Average value = $\frac{1}{2-(-2)}\int_{-2}^{2}(1-x^2)\,dx$

$$= \frac{1}{4}\left[x-\frac{1}{3}x^3\Big|_{-2}^{2}\right]$$

$$= \frac{1}{4}\left[\left(2-\frac{8}{3}\right)-\left(-2+\frac{8}{3}\right)\right]$$

$$= \frac{1}{4}\left[4-\frac{16}{3}\right]$$

Find the average value: let $y = 3x^2$ on $[0,3]$

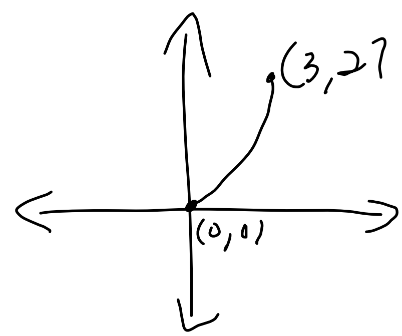
Our formula is Average value $= \frac{1}{b-a}\int_a^b f(x)\,dx$

$$\frac{1}{3-0}\int_0^3 3x^2\,dx$$

$$= \frac{1}{3}\left[x^3\Big|_0^3\right]$$

$$= \frac{1}{3}[27-0]$$

$$= 9$$



Find the average value: let $y = 2\cos x$ on $\left[0, \frac{\pi}{2}\right]$

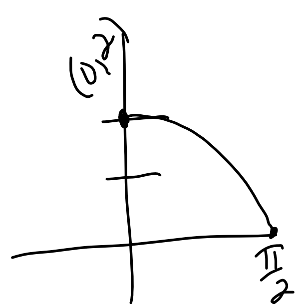
Our formula is Average value $= \frac{1}{b-a}\int_a^b f(x)\,dx$

$$\frac{1}{\frac{\pi}{2}-0}\int_0^{\frac{\pi}{2}} 2\cos x\,dx$$

$$= \frac{2}{\pi}\left[2\sin x\Big|_0^{\frac{\pi}{2}}\right]$$

$$= \frac{2}{\pi}[2-0]$$

$$= \frac{4}{\pi}$$



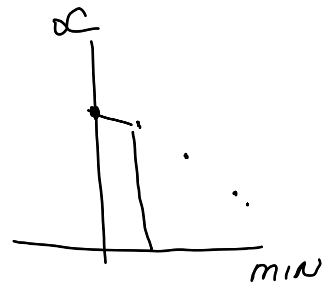
Now that we have found the average value, let's find the average rate of change for $y = 2\cos x$ on $\left[0, \frac{\pi}{2}\right]$

$$\text{A R of } \Delta \text{ on } \left[0, \tfrac{\pi}{2}\right] = \frac{f\left(\frac{\pi}{2}\right) - f(0)}{\frac{\pi}{2} - 0}$$

$$= \frac{0 - 2}{\frac{\pi}{2}}$$

$$= -\frac{4}{\pi}$$

Now let's look at some problems that ask for average value.



1. 2011 AB2 [calculator]

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

$\frac{1}{10}\int_0^{10} H(t)\,dt$ gives us the AVERAGE TEMP IN °C OF THE TEA DURING THE TIME INTERVAL $0 \le t \le 10$ MINUTES

$$\int_0^{10} H(t)\,dt \approx \text{TRAP}$$

$$\text{TRAP} = \frac{H(0)+H(2)}{2}(2) + \frac{H(2)+H(5)}{2}(3) + \frac{H(5)+H(9)}{2}(4)$$

$+ \frac{H(9)+H(10)}{2}(1)$

$= 529.5\ (°C)(min)$

AV TEMP $= \frac{1}{10}[529.5]$

$= 52.95°C$

2. *2010 AB2 [calculator]*

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon $(t = 0)$ and 8 P.M. $(t = 8)$. The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$.

Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

$\frac{1}{8}\int_0^8 E(t)\,dt$ gives us the AVERAGE # OF entries IN HUNDREDS for the contest for the Time INTERVAL $0 \le t \le 8$ hours or for the Time interval from noon TO 8 pm

$\int_0^8 E(t)\,dt \approx$ TRAP

$$\text{TRAP} = \frac{E(0)+E(2)}{2}(2) + \frac{E(2)+E(5)}{2}(3) + \frac{E(5)+E(7)}{2}(2) + \frac{E(7)+E(8)}{2}(1)$$

$= 85.5$

AV # OF entries [IN HUNDREDS] $= \frac{1}{8}[85.5]$

≈ 10.688 or 10.689

Thomasville, Oregon Problem

Traffic engineers will consider turn restrictions when $L(t) \geq 150$ cars per hour. Find all values of t such that $L(t) \geq 150$ and compute the average value of L over this time interval. Indicate units of measure.

$$L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$$

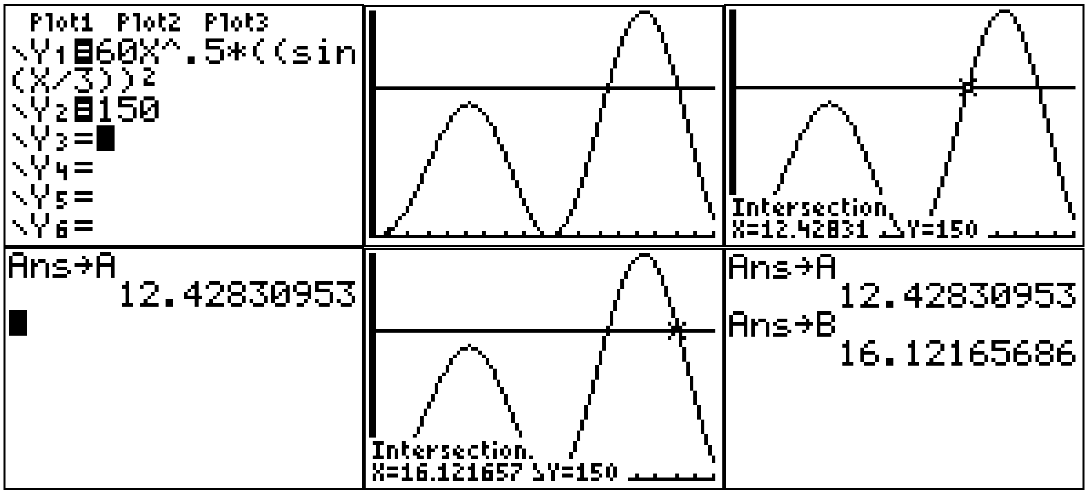
$150 = L(t)$ at

$t \approx 12.42831$ call it "A"

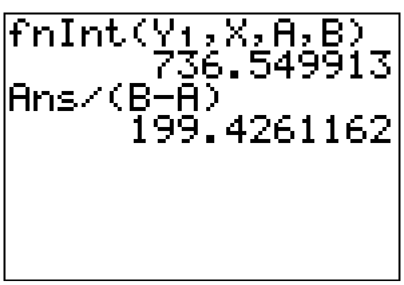
$t \approx 16.12166$ call it "B"

$$\frac{1}{B-A}\int_A^B L(t)\,dt$$

av # of $\frac{\text{cars}}{\text{hour}} \approx 199.426$



Now we know our interval so we can find the average value using our handy-dandy formula



4. 2008 AB2 [calculator]

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

Homework: page 291 #47 – 50 [Just find the average value!!!]