Today's Announcements:

Calculus the Musical on Monday at 7 pm
Free
At Monarch High School

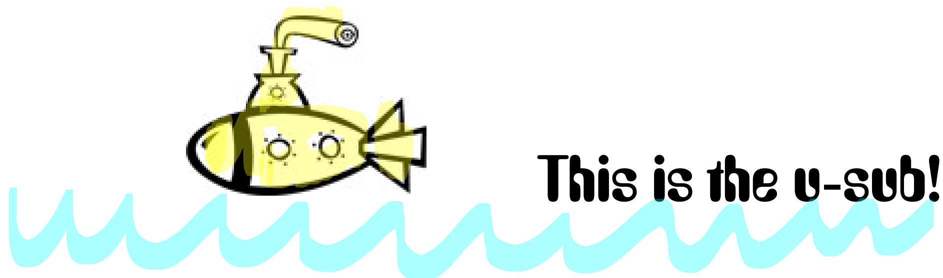
T-shirts $7 cash only

Quiz on Friday the 13th [next Friday]
Multiple-choice only this time

We will be working on a u-project tomorrow

COST =
$12,000,000

Using u-sub with Definite Integrals



There are two schools of thought on how to do this. I am an advocate of re-writing everything, including the upper and lower bounds, in terms of *u* and evaluating in terms of *u*. This method actually saves time.

Example

$$\int_2^5 f'(3x)\,dx$$

$u = 3x$ $\qquad u(2) = 6$

$du = 3\,dx$ $\qquad u(5) = 15$

$\frac{1}{3}du = dx$

$$= \frac{1}{3}\int_6^{15} f'(u)\,du$$

$$= \frac{1}{3} f(u) \Big|_6^{15}$$

$$= \frac{1}{3} f(15) - \frac{1}{3} f(6)$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 2\sin(2x)\,dx$$ Let $u = 2x$

Then $du = 2\,dx$

But our upper and lower bounds are $x-$values. We can re-write the bounds and then no longer worry about any $x-$values.

$u\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$ Our new lower bound

$u\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$ Our new upper bound

We can re-write our integral in terms of u

$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 2\sin(2x)\,dx$ will be rewritten as:

Since we replaced everything, we can now forget about "x"

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin u\,du$$

$$= -\cos u \Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= 0 - \left(-\frac{1}{2}\right)$$

$$= \frac{1}{2}$$

Another Example:

$$\int_0^1 3x^2\left(x^3+1\right)^4 dx$$

$u = x^3+1$

$du = 3x^2\,dx$

$u(0) = 1$

$u(1) = 2$

$$= \int_1^2 u^4\,du$$

$$= \left.\frac{u^5}{5}\right|_1^2$$

$$= \frac{32}{5} - \frac{1}{5}$$

Try:

$$\int_{-1}^0 x\sqrt{1-x^2}\,dx$$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}\,du = x\,dx$

$u(-1) = 0$

$u(0) = 1$

$$= -\frac{1}{2}\int_0^1 u^{\frac{1}{2}}\,du$$

$$= -\frac{1}{2}\left.\left(\frac{2}{3}u^{\frac{3}{2}}\right)\right|_0^1$$

$$= -\frac{1}{3}(1-0)$$

$$= -\frac{1}{3}$$

$$\int_0^{\frac{\pi}{6}} \cos(2x)\, dx$$

$u = 2x$

$du = 2\, dx$

$\frac{1}{2} du = dx$

$u(0) = 0$

$u\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$

$$= \frac{1}{2}\int_0^{\frac{\pi}{3}} \cos u\, du$$

$$= \frac{1}{2} \sin u \Big|_0^{\frac{\pi}{3}}$$

$$= \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)$$

$$\int_0^3 \frac{1}{\sqrt{x+1}}\, dx$$

$u = x + 1$

$du = dx$

$u(0) = 1$

$u(3) = 4$

$$= \int_1^4 u^{-\frac{1}{2}}\, du$$

$$= 2\sqrt{u}\Big|_1^4$$

$$= 2(\sqrt{4} - \sqrt{1})$$

$$= 2$$

$$\int_0^{\frac{\pi}{4}} \sin^2 x \cos x \, dx$$

$$u = \sin x \quad u(0) = 0$$
$$du = \cos x \, dx \quad u\left(\tfrac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$= \int_0^{\frac{\sqrt{2}}{2}} u^2 \, du$$

$$= \frac{1}{3} u^3 \Big|_0^{\frac{\sqrt{2}}{2}}$$

$$= \frac{1}{3}\left(\frac{\sqrt{2}}{2}\right)^3 = \frac{1}{3}\left(\frac{2\sqrt{2}}{8}\right)$$
$$= \frac{\sqrt{2}}{12}$$

$$\int_{\frac{\pi^2}{36}}^{\frac{\pi^2}{4}} \frac{\cos\sqrt{x}}{\sqrt{x}} \, dx$$

$$u = \sqrt{x} \quad u\left(\tfrac{\pi^2}{36}\right) = \frac{\pi}{6}$$
$$du = \frac{1}{2\sqrt{x}} \, dx \quad u\left(\tfrac{\pi^2}{4}\right) = \frac{\pi}{2}$$
$$2\,du = \frac{1}{\sqrt{x}} \, dx$$

$$= 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos u \, du$$

$$= 2 \sin u \Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= 2\sin\frac{\pi}{2} - 2\sin\frac{\pi}{6}$$

$$= 2 - 1$$

$$= 1$$

$$\int_1^3 3\, g'(3x)\, dx$$

$u = 3x \qquad u(1) = 3$

$du = 3\,dx \qquad u(3) = 9$

$$= \int_3^9 g'(u)\, du$$

$$= g(u) \Big|_3^9$$

$$= g(9) - g(3)$$

Homework: page 305 # 71, 73, 75, 76, 81 [you must show all steps with proper notation]