HAPPY NEW YEAR!

CALCULUS THE MUSICAL NEXT MONDAY
7 PM
FREE
AT MONARCH HIGH SCHOOL

T-SHIRTS = $7 CASH ONLY

Chapter four multiple-choice quiz next Friday

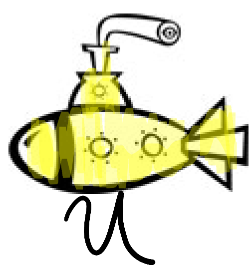
Integration by Substitution [Anti-Chain Rule]

We will call this "u-sub"

$$\frac{d}{dx}[f(g(x))] = f'(g(x))\,g'(x)$$

$$\int f'(g(x))\,g'(x)\,dx = f(g(x)) + C$$

You should look for a composite function! Just like the Chain Rule, we'll need to find a u.



This is the u-sub!

Function	What is u?	What is $\frac{du}{dx}$?
$2\sin(2x)$	$2x$	2
$(3x^7+11)^5(21x^6)$	$3x^7+11$	$21x^6$
$\sqrt{4x^5-5x}\,(20x^4-5)$	$4x^5-5x$	$20x^4-5$
$\tan^2 x\sec^2 x$ $(\tan x)^2(\sec x)^2$	$\tan x$	$\sec^2 x$

Here is our first example:

$$\int \left(x^2+5\right)^{10}(2x)\,dx$$

$$u = x^2+5$$

$$\frac{du}{dx} = 2x$$

$$du = 2x\,dx$$

$$= \int u^{10}\,du$$

$$= \frac{1}{11}u^{11} + C$$

$$= \frac{1}{11}\left(x^2+5\right)^{11} + C$$

Our second example:

$$\int 2\sin(2x)\,dx$$

$$u = 2x$$

$$\frac{du}{dx} = 2$$

$$du = 2\,dx$$

$$= \int \sin u\,du$$

$$= -\cos u + C$$

$$= -\cos(2x) + C$$

$$\int \sqrt{4x^5 - 5x}\,(20x^4 - 5)\,dx$$

$u = 4x^5 - 5x$

$du = (20x^4 - 5)\,dx$

$$= \int u^{\frac{1}{2}}\,du$$

$$= \frac{2}{3}u^{\frac{3}{2}} + C$$

$$= \frac{2}{3}(4x^5 - 5x)^{\frac{3}{2}} + C$$

In general, integration by u-sub looks like:

$$\int (\textit{composite function})(\textit{another function}^*)\,dx$$

*The other function looks like the derivative of the "inside" of the composite function. Look for the composite function!

Try:

$$\int \frac{10x}{(5x^2+7)^3}\,dx$$

$u = 5x^2 + 7$

$du = 10x\,dx$

$$= \int u^{-3}\,du$$

$$= -\frac{1}{2}u^{-2} + C$$

$$= -\frac{1}{2}(5x^2+7)^{-2} + C$$

$$\int \tan^2 x \sec^2 x\,dx$$

$u = \tan x$

$du = \sec^2 x\,dx$

$$= \int u^2\,du$$

$$= \frac{1}{3}u^3 + C$$

$$= \frac{1}{3}\tan^3 x + C$$

What if we do not quite have du?

$\int x^3 (x^4+3)^2 \, dx$

$u = x^4 + 3$

$du = 4x^3 \, dx$

$\frac{1}{4} du = x^3 \, dx$

$= \frac{1}{4} \int u^2 \, du$

$= \frac{1}{4}\left(\frac{1}{3}u^3\right) + C$

$= \frac{1}{12}(x^4+3)^3 + C$

$\int \cos(3x) \, dx$

$u = 3x$

$du = 3 \, dx$

$\frac{1}{3} du = dx$

$= \frac{1}{3} \int \cos u \, du$

$= \frac{1}{3} \sin u + C$

$= \frac{1}{3} \sin(3x) + C$

$\int 2x^3 \sqrt{x^4+5} \, dx$

$u = x^4 + 5$

$du = 4x^3 \, dx$

$\frac{1}{2} du = 2x^3 \, dx$

$= \frac{1}{2} \int u^{\frac{1}{2}} \, du$

$= \frac{1}{2}\left(\frac{2}{3} u^{\frac{3}{2}}\right) + C$

$= \frac{1}{3}(x^4+5)^{\frac{3}{2}} + C$

Try:

$\int x(x^2+3)^9\,dx$

$u = x^2+3$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int u^9\,du$

$= \frac{1}{2}\left(\frac{1}{10}u^{10}\right)+C$

$= \frac{1}{20}(x^2+3)^{10}+C$

$\int \frac{x^5+x^3}{x^2}\,dx$ Do we need u-substitution? NO

$= \int (x^3+x)\,dx$ JUST SIMPLIFY

$\int \frac{3x^2}{(x^3+1)^4}\,dx$ Do we need u-substitution? yes

$u = x^3+1$

$du = 3x^2\,dx$

$\int \sin^2 \theta \cos \theta \; d\theta$ Do we need u-substitution? yes

$u = \sin \theta$

$du = \cos \theta \, d\theta$

$\int \left(\sin^2 \theta + \cos^2 \theta \right) d\theta$ Do we need u-substitution? NO

$= \int 1 \, d\theta$

Here is a different looking one!

$\int 5x^4 \, g'\left(x^5\right) dx$

$u = x^5$

$du = 5x^4 \, dx$

$= \int g'(u) \, du$

$= g(u) + C$

$= g(x^5) + C$

Homework: page 304 #7, 9, 13, 19, 21, 23
Show ALL steps