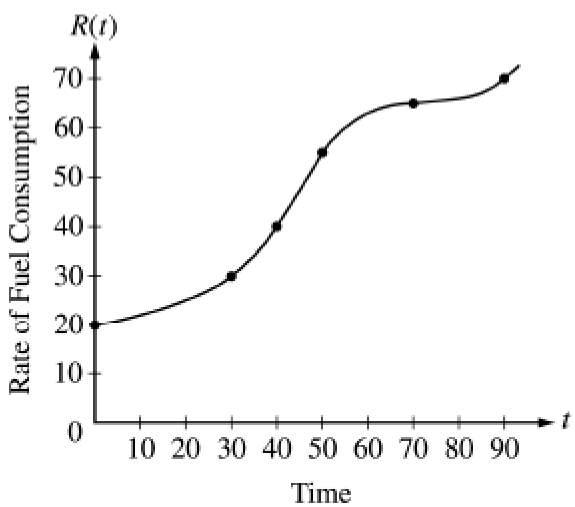
Trapezoid Rule-another geometric approach to definite integrals

-used when given data or used when we don't know the anti-derivative of the integrand
-similar to Rectangular Approximation Method [RAM] but uses trapezoids not rectangles [hence the name]
- intervals may or may not have same width

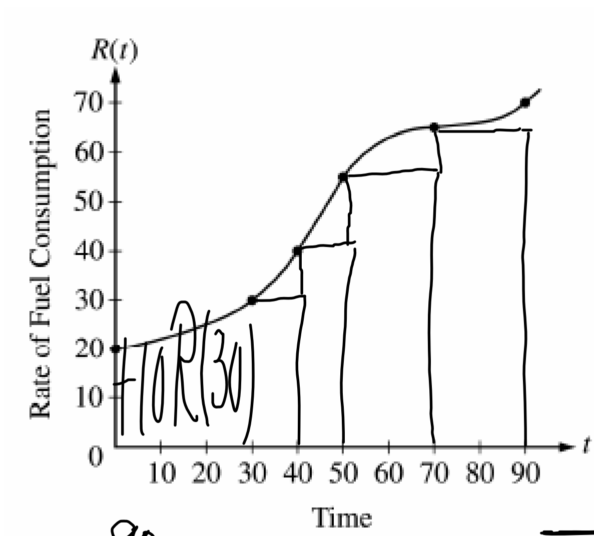
The airplane problem!

2003 AB 3 [calculator-friendly]



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.



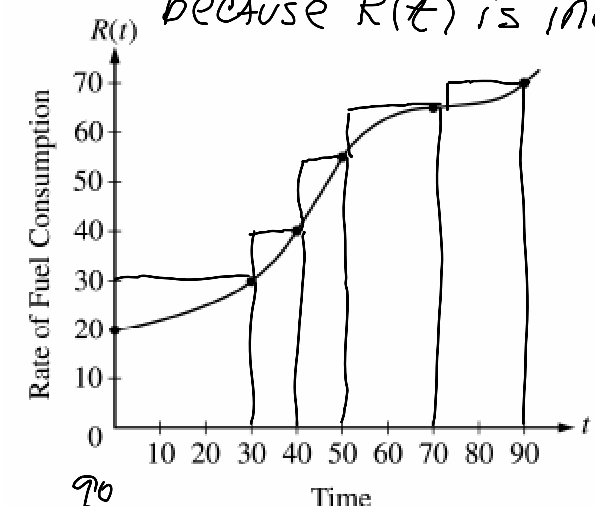
t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

$\int_0^{90} R(t)\,dt \approx LRAM$

$LRAM = 30R + 10R(40) + 20R(50) + 20R(70)$

≈ 3700 gallons

this APPROXIMATION is less than the ACTUAL VALUE because $R(t)$ is increasing on $0 \le t \le 90$ minutes

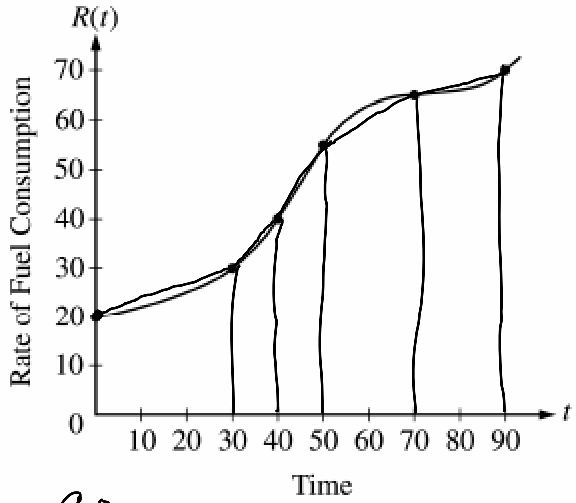


t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

$\int_0^{90} R(t)\,dt \approx RRAM$

$RRAM = 30R(30) + 10R(40) + 10R(50) + 20R(70) + 20R(90)$

≈ 4550 gallons



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

$$\int_0^{90} R(t)\,dt \approx \text{TRAP}$$

$$\text{TRAP} = \frac{R(0)+R(30)}{2}(30) + \frac{R(30)+R(40)}{2}(10) + \frac{R(40)+R(50)}{2}(10) + \frac{R(50)+R(70)}{2}(20) + \frac{R(70)+R(90)}{2}(20)$$

$$= 4125 \text{ gallons}$$

Is LRAM an over- or under-estimate? UNDER

Is RRAM an over- or under-estimate? OVER

Is TRAP an over- or under-estimate? hard TO say

Which is the best estimate? TRAPEZOID

Why? FITS CURVE BETTER

What do you obtain when you take the average of LRAM and RRAM? $\frac{3700+4550}{2} = 4125$

Example 2

Using the table, estimate the total distance traveled from time $t = 0$ to $t = 6$ using LRAM, RRAM, MRAM, and the Trapezoid rule

Time, t	0	1	2	3	4	5	6
Velocity, $v(t)$	3	4	5	4	7	8	11

Since $v(t) > 0$ on this interval, then the total distance traveled can be found by $\int_0^6 v(t)\,dt$

Now let's estimate using LRAM, RRAM, MRAM, and try using trapezoids. Notice that the intervals are evenly spaced.

$$\int_0^6 v(t)\,dt \approx LRAM$$

LRAM $= 1\left[v(0)+v(1)+v(2)+v(3)+v(4)+v(5)\right]$

$= 31$

$$\int_0^6 v(t)\,dt \approx RRAM$$

RRAM $= 1\left[v(1)+v(2)+v(3)+v(4)+v(5)+v(6)\right]$

RRAM = 39

$$\int_0^6 v(t)\,dt \approx MRAM$$

MRAM= $2\left[v(1)+v(3)+v(5)\right]$

$= 32$

$$\int_0^6 v(t)\,dt \approx TRAP$$

Let's sum up the areas of each trapezoid and look for a pattern.

$$TRAP = \frac{v(0)+v(1)}{2}(1) + \frac{v(1)+v(2)}{2}(1) + \frac{v(2)+v(3)}{2}(1)$$
$$+ \frac{v(3)+v(4)}{2}(1) + \frac{v(4)+v(5)}{2}(1) + \frac{v(5)+v(6)}{2}(1)$$

Let's simplify this so that the computation is easier.

$$TRAP = \frac{1}{2}\left[v(0)+2v(1)+2v(2)+2v(3)+2v(4)+2v(5)+v(6)\right]$$

$$= 35$$

Hmm! Isn't 35 the average of LRAM and RRAM?
There is a formula that you memorize but it is probably just as easy to find the sum of the trapezoid areas. Here is the formula:

Let f be continuous on $[a,b]$. The Trapezoid Rule for approximating $\int_a^b f(x)\,dx$ is given by:

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left[f(x_0)+2f(x_1)+2f(x_2)+\ldots+2f(x_{n-1})+f(x_n)\right]$$

Where n is the number of trapezoids, a is the lower bound, and b is the upper bound. THE INTERVALS MUST BE EVENLY SPACED!!!
In our example above, $a=0,\ b=6,\ n=6$

Always be very careful when given data in a table. If you do not have equal subdivisions, then the above formula does not work! If this is the case, then just find the sum of the areas of the trapezoids.

Consider the table of velocity values below and estimate the total distance traveled using the Trapezoid Rule. [Note: $v(t)>0$ for the interval.

t(sec)	0	1	5	6	8
$v(t)$ $m/$sec	0	2	3	5	9

$$TDT = \int_0^8 v(t)\,dt \approx TRAP$$

We cannot use the formula because we have unequal subdivisions.

$$TRAP = \frac{v(0)+v(1)}{2}(1) + \frac{v(1)+v(5)}{2}(4) + \frac{v(5)+v(6)}{2}(1) + \frac{v(6)+v(8)}{2}(2)$$

$$= 29 \text{ meters}$$

2009 AB6B [non-calculator]

Question 6

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

(b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

$$\int_{20}^{40} v(t)\,dt \approx \text{TRAP}$$

$$\text{TRAP} = \frac{v(20)+v(25)}{2}(5) + \frac{v(25)+v(32)}{2}(7) + \frac{v(32)+v(40)}{2}(8)$$

$$= -75 \text{ m}$$

$\int_{20}^{40} v(t)\,dt$ is the PARTICLE'S CHANGE IN POSITION IN METERS FROM TIME $t = 20$ SEC TO TIME $t = 40$ SEC

Homework: page 315 #51, 52 AND page 320 10d
As always, show all steps
[Do NOT just write down a bunch of numbers]

IF YOU CAN [OR FEEL LIKE IT] USE THE FORMULA