The Ultimate* Particle Motion Problem–that uses a table

* in my humble opinion

Harry Potter rides a thestral back and forth along a straight path in the Forbidden Forest.

The table below gives values for the velocity and acceleration of the pair for selected values of time t. Both velocity and acceleration are differentiable functions of time t. Use the data in the table to answer the following problems.

Time, t [seconds]	0	10	30	60
$v(t)$ $m/\sec$	10	-20	20	40
$a(t)$ $m/\sec^2$	0	-10	10	-20

(A) Use a Right Riemann Sum to estimate $\int_0^{60} v(t)\,dt$. Explain the meaning of the definite integral in the context of this problem. Indicate units.

$\int_0^{60} v(t)dt$ gives us the change in position in meters during $0 \le t \le 60$ sec

RRAM $= 10v(10) + 20v(30) + 30v(60)$

$= 1400$ m

(B) Approximate the total distance traveled for $0 \le t \le 60$ using a trapezoidal sum. Indicate units.

$$TDT = \int_0^{60} |v(t)|\,dt \approx TRAP$$

$$TRAP = \frac{|v(0)| + |v(10)|}{2}(10) + \frac{|v(10)| + |v(30)|}{2}(20) + \frac{|v(30)| + |v(60)|}{2}(30)$$

$$= 1450 \text{ m}$$

(C) Is the speed increasing at time $t = 10$? Justify.

$v(10) = -20$ m/sec $\quad a(10) = -10$ m/sec^2

at $t = 10$ $v(t) < 0$ and $a(t) < 0$

Hence speed is increasing at $t = 10$

(D) Is the speed increasing at time $t = 30$? Justify.

$v(30) = 20$ m/sec $\quad a(30) = 10$ m/sec^2

at $t = 30$ $a(t)$ and $v(t) > 0$ Hence

Speed is increasing

(E) Is the speed increasing at time $t = 60$? Justify.

$v(60) = 40$ m/sec $\quad a(60) = -20$ m/sec^2

at $t = 10$ $v(t) > 0$ and $a(t) < 0$

So speed is NOT increasing at $t = 60$

(F) Determine the value of $\int_0^{60} a(t)\,dt$. Indicate units and explain the meaning of the definite integral in the context of this problem.

$$\int_0^{60} a(t)\,dt$$

$$= v(t)\Big|_0^{60}$$

$$= v(60) - v(0)$$

$$= 30 \text{ m/sec}$$

$a(t) = v'(t)$

$\int_0^{60} v'(t)\,dt$ gives us the change in velocity in m/sec over $0 \le t \le 60$ sec

(G) Given that the initial position of the thestral and Harry is at 6 meters, find the position of the thestral at time $t = 30$. [Set-up only]

$$x(30) = x(0) + \int_0^{30} v(t)\,dt$$

INITIAL POSITION + CHANGE IN POSITION over $0 \le t \le 30$

(H) Given that the initial position of the thestral and Harry is at 6 meters, find the position of the thestral at time $t = 60$. [Set-up only]

$$x(60) = x(0) + \int_0^{60} v(t)\,dt$$

(I) Find the average acceleration over the time interval $0 \le t \le 60$

$$\text{av acc over } [0,60] = \frac{v(60)-v(0)}{60-0} = \frac{1}{2}\ \frac{m}{sec^2}$$

(J) Use a Left Riemann Sum to estimate the average velocity over the time interval $0 \le t \le 60$

$$\text{Av vel over } [0,60] = \frac{1}{60-0}\int_0^{60} v(t)\,dt$$

$$\int_0^{60} v(t)\,dt \approx LRAM \quad LRAM = 10v(0)+20v(10)+30v(30) = 300m$$

$$\text{Av vel} = \frac{300\,m}{60\,sec} = 5\,\frac{m}{sec}$$

(K) Set up but do not evaluate an integral that finds the average speed during the time interval $0 \le t \le 60$

$$\text{Av speed over } [0,60] = \frac{1}{60-0}\int_0^{60} |v(t)|\,dt$$

(L) At time $t = 10$ is the pair moving left or right? Justify.

$v(10) = -20\ m/sec \quad v(10) < 0$

Hence moving left

(M) At time $t = 60$ is the pair moving left or right? Justify.

$v(60) = 40$ m/sec $\quad v(60) > 0$

Hence moving right