LIMITS [The analytic approach]

What we know so far about limits:

If $f(x)$ is a "well-behaved" function, then

$$\lim_{x \to c} f(x) = f(c)$$

This is called the direct substitution method.

A limit ***does not exist*** *at* $x = c$ ["dne"] if we have either a jump discontinuity, oscillating behavior at $x = c$, or an infinite discontinuity. [Notice that a limit DOES occur if there is a removable discontinuity]

QUESTION: If $\lim_{x\to c} f(x) = L$, then does $f(c) = L$?

If $\lim_{x\to 2} f(x) = 5$, does $f(2) = 5$?

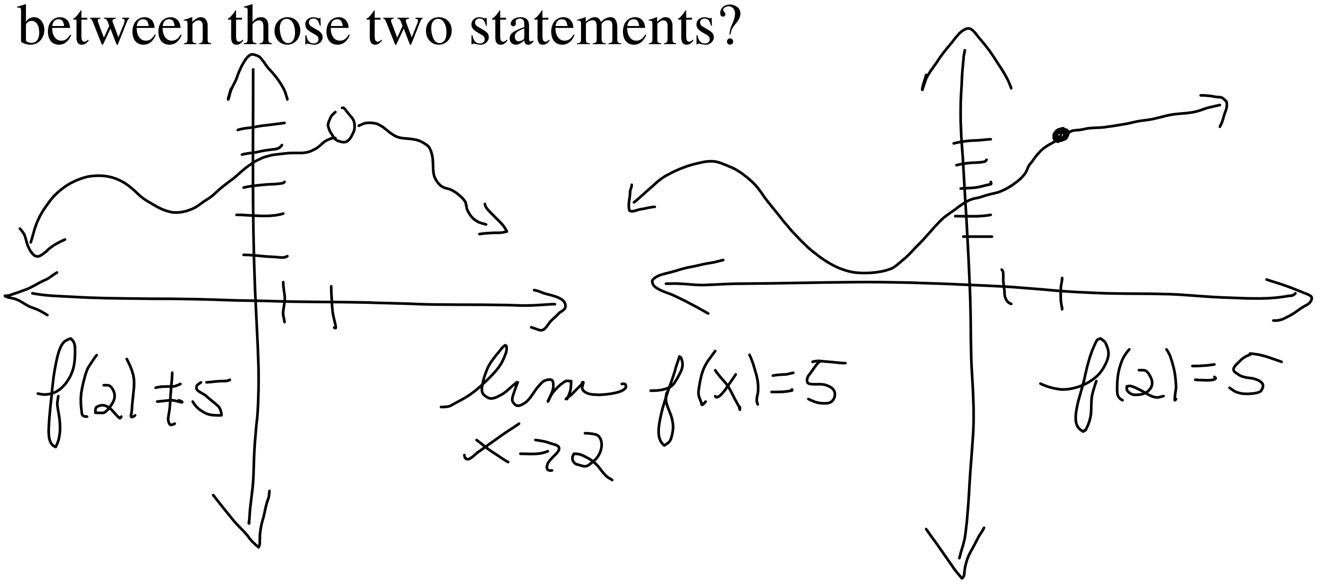
NO

Is this the same as,

Given that $f(x)$ is a continuous function, if $\lim_{x\to 2} f(x) = 5$, does $f(2) = 5$?

yes

Can you draw two graphs that show the difference between those two statements?

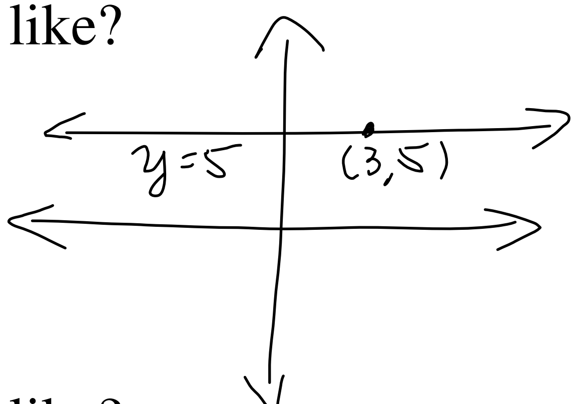


Some Basic Limit Properties

(1) $\lim\limits_{x\to c} b = b$ where $b, c \in$ real numbers

What does this look like?

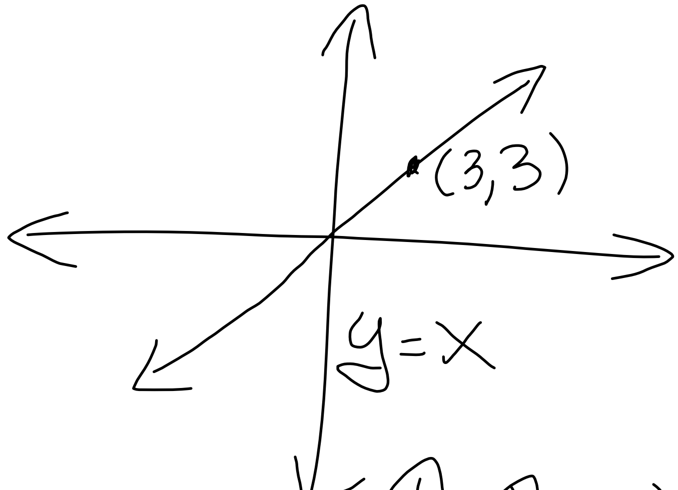
$\lim\limits_{x\to 3} 5 =$ 5



(2) $\lim\limits_{x\to c} x = c$

What does this look like?

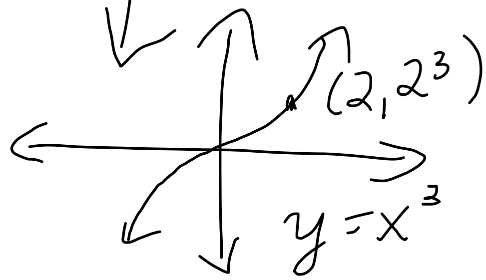
$\lim_{x\to 3} x = 3$



(3) $\lim_{x\to c} x^n = c^n$

What does this look like?

$\lim_{x\to 2} x^3 = 2^3$



See page 59 for Theorem 1.2 ***Properties of Limits***
[also pages 60, 61]

A LOOK AT THE BASIC PROPERTIES

Examples below

Given that: $\lim_{x\to 7} f(x) = 10$ ***and*** $\lim_{x\to 7} g(x) = 25$

Using the Scalar multiple property

$\lim_{x\to 7} 10 f(x)$

$= 10\left[\lim_{x\to 7} f(x)\right]$

$= 100$

Sum or difference property

$\lim_{x\to 7} [f(x) - g(x)]$

$= \left[\lim_{x\to 7} f(x)\right] - \left[\lim_{x\to 7} g(x)\right]$

$= 10 \quad -25$

$= -15$

Product Property

$$\lim_{x\to 7}[f(x)g(x)]$$

$$= \left[\lim_{x\to 7} f(x)\right]\left[\lim_{x\to 7} g(x)\right]$$

$$= (10)(25)$$

$$= 250$$

Quotient Property

$$\lim_{x\to 7}\frac{f(x)}{g(x)}$$

$$= \frac{\lim_{x\to 7} f(x)}{\lim_{x\to 7} g(x)}$$

$$= \frac{10}{25}$$

Power Property

$$\lim_{x\to 7}[g(x)]^2$$

$$= \left[\lim_{x\to 7} g(x)\right]^2$$

$$= 25^2$$

Note: You don't need to use the properties if you can just do direct substitution

Given: $\lim_{x \to c} f(x) = \frac{3}{2}$ and $\lim_{x \to c} g(x) = \frac{1}{2}$

(1) $\lim_{x \to c} [4 f(x)]$

$= 4\left[\lim_{x \to c} f(x)\right]$ SCALAR

$= 6$

(2) $\lim_{x \to c} [f(x) + g(x)]$

$= \left[\lim_{x \to c} f(x)\right] + \left[\lim_{x \to c} g(x)\right]$ SUM

$= 2$

(3) $\lim_{x \to c} [f(x)\, g(x)]$

$= \left[\lim_{x \to c} f(x)\right]\left[\lim_{x \to c} g(x)\right]$ PRODUCT

$= \frac{3}{4}$

(4) $\lim_{x \to c} \frac{f(x)}{g(x)}$ Quotient

$= \dfrac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = 3$

(5) $\lim_{x \to c} (g(x))^2$ POWER

$= \left[\lim_{x \to c} g(x)\right]^2$

$= \frac{1}{4}$

These properties also work for trigonometric functions too!

$$\lim_{x \to \frac{\pi}{2}} x \sin x$$

COULD USE PRODUCT BUT EASIER WITH DIRECT SUB.

$= \frac{\pi}{2}$

♪ If direct substitution works, then you do not need to use the properties.

All polynomial limits can be found with direct substitution. All continuous function limits can also be found with direct substitution. This includes finding the limit of a composite function.

Let's find some limits either with direct substitution or with properties
Page 67: # 22, 24, 40[not homework!]

22. $\lim_{x \to 0} (2x-1)^3$

$= -1$

JUST USE DIRECT SUB.

24. $f(x) = x + 7 \qquad g(x) = x^2$

(a) $\lim_{x \to -3} f(x)$

$= -3 + 7$

$= 4$

(b) $\lim_{x \to 4} g(x)$

$= 4^2$

$= 16$

(c) $\lim_{x \to -3} g(f(x))$

$= \lim_{x \to -3} g(f(-3))$

$= \lim_{x \to -3} g(4)$

$= 16$

40. $\lim_{x \to c} f(x) = 27$

(a) $\lim_{x \to c} \sqrt[3]{f(x)}$ POWER

$= \left[\lim_{x \to c} f(x)\right]^{\frac{1}{3}}$

$= 27^{\frac{1}{3}}$

(b) $\lim_{x \to c} \frac{f(x)}{18}$ SCALAR

$= \frac{1}{18}\left[\lim_{x \to c} f(x)\right]$

$= \frac{27}{18}$

(c) $\lim_{x \to c} \left[f(x)\right]^2$ POWER

$= \left[\lim_{x \to c} f(x)\right]^2$

$= 27^2$

(d) $\lim_{x \to c} \left[f(x)\right]^{2/3}$ POWER

$= \left[\lim_{x \to c} f(x)\right]^{\frac{2}{3}}$

$= 27^{\frac{2}{3}}$

Homework: page 67: # 23, 24, 25, 26, 29, 31, 37, 39

Show steps

use STANDARD notation