Fundamental Theorem of Calculus Handout 2011ftc

Accumulated [or total or net] change is given by the definite integral whose integrand is the rate of change.

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

Think: Change in f from $x = a$ to $x = b$

Consider these examples:

Exercises

Write a sentence to answer each of the following questions.

1. If $h(t)$ is the rate of change of the height of a conical pile of sand measured in feet per hour, what does $\int_0^5 h(t)\,dt$ represent? Answer in correct units.

2. If $v(t)$ is the velocity of a particle moving along the x-axis, measured in feet per second, what does $\int_3^{10} v(t)\,dt$ represent? Answer in correct units.

3. If $b(t)$ is the rate of growth of the number of bacteria in a dish, measured in number of bacteria per hour, what does $\int_2^6 b(t)\,dt$ represent? Answer in correct units.

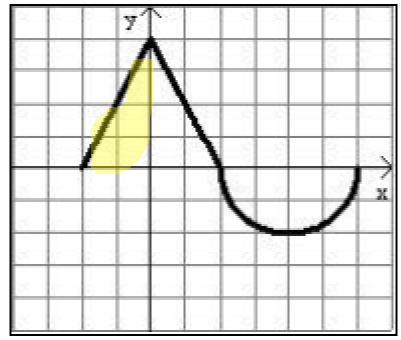
4. If $v(t)$ is the velocity of a particle moving along the x-axis at time t, and the position $x(t)$ is 5 at time $t = 2$, (a) write an integral expression that represents the position of the particle at time $t = 10$, and (b) write an integral expression that gives the total distance traveled by the particle from time $t = 2$ to time $t = 10$.

5. If $p(t)$ is the rate of growth of a rabbit population, measured in rabbits per year, and there were 100 rabbits in the year 2005 ($t = 0$), write an integral expression that represents the rabbit population in 2007.

Now for a geometric look at FTC

The graph of f' on $-2 \le x \le 6$ consists of two line segments and a semicircle as shown at right. Given that $f(-2) = 5$,

find $f(0)$, $f(2)$, and $f(6)$.



Graph of f'

$$f(0) = f(-2) + \int_{-2}^{0} f'(x)\,dx$$

INITIAL VALUE + ACCUMULATED RATE OF CHANGE

algebraic LOOK

$$f(0) = f(-2) + [f(0) - f(-2)]$$

NUMERICALLY

$$f(0) = 5 + \tfrac{1}{2}(2)(4) = 9$$

$$f(2) = f(-2) + \int_{-2}^{2} f'(x)\,dx$$

$$= 5 + \tfrac{1}{2}(4)(4) = 13$$

$$f(6) = f(-2) + \int_{-2}^{6} f'(x)\,dx$$

$$= 5 + 8 - \tfrac{1}{2}\pi(2^2) \text{ or } 13 - 2\pi$$

Here is a nice calculator-friendly FTC problem to ponder

A pizza with a temperature of 95°C is put into a 25°C room when $t = 0$. The pizza's temperature is decreasing at a rate of $r(t) = 6e^{-0.1t}$ °C per minute. Estimate the pizza's temperature when $t = 5$ minutes.

PIZZA is 95°C at time = 0

INITIAL VALUE

$$95 - \int_0^5 r(t)\,dt$$

$$\approx 71.392°C$$

Another calculator-friendly problem

A particle moving along the x-axis has position $x(t)$ at time t with the velocity of the particle given by $v(t) = 5\sin(t^2)$ At time $t = 6$, the particle's position is (6, 0). Find the position of the particle when $t = 7$.

$v(t) = x'(t)$

? I think that the position should be (6, 0)

$$x(7) = x(6) + \int_6^7 v(t)\,dt$$

$$x(7) \approx -.163$$

And for the first of many water-related problems

Water flows into a tank at a rate of $\frac{dW}{dt} = \frac{1}{75}\left(600 + 20t - t^2\right)$ where $\frac{dW}{dt}$ is measured in gallons per hour and t is measured in hours. If there are 150 gallons of water in the tank at time $t = 0$, how many gallons of water are in the tank when $t = 24$?

think: $\frac{dW}{dt} = W'(t)$

$W(0) = 150$ gallons

$$W(24) = W(0) + \int_0^{24} W'(t)\,dt$$

$$W(24) = 357.36$$