

Accumulated [or total or net] change is given by the definite integral whose integrand is the rate of change.

$$\int_a^b f'(x) dx = f(b) - f(a)$$

Think: Change in f from $x = a$ to $x = b$

Consider these examples:

Exercises

Write a sentence to answer each of the following questions.

1. If $h(t)$ is the rate of change of the height of a conical pile of sand measured in feet per hour, what does $\int_0^5 h(t) dt$ represent? Answer in correct units.

CHANGE in height of cone in feet during $0 \leq t \leq 5$ hours

2. If $v(t)$ is the velocity of a particle moving along the x -axis, measured in feet per second, what does $\int_3^{10} v(t) dt$ represent? Answer in correct units.

DISPLACEMENT OR THE NET CHANGE in position, in feet, during $3 \leq t \leq 10$ seconds

3. If $b(t)$ is the rate of growth of the number of bacteria in a dish, measured in number of bacteria per hour, what does $\int_2^6 b(t) dt$ represent? Answer in correct units.

CHANGE IN NUMBER OF BACTERIA
DURING $2 \leq t \leq 6$ HOURS

4. If $v(t)$ is the velocity of a particle moving along the x -axis at time t , and the position $x(t)$ is 5 at time $t = 2$, (a) write an integral expression that represents the position of the particle at time $t = 10$, and (b) write an integral expression that gives the total distance traveled by the particle from time $t = 2$ to time $t = 10$.

$$x(2) = 5$$

INITIAL POSITION + DISPLACEMENT

$$x(10) = x(2) + \int_2^{10} v(t) dt$$

$$x(10) = x(2) + \int_2^{10} x'(t) dt$$

$$v(t) = x'(t)$$

$$(b) \text{ TDT on } [2, 10] = \int_2^{10} |v(t)| dt$$

5. If $p(t)$ is the rate of growth of a rabbit population, measured in rabbits per year, and there were 100 rabbits in the year 2005 ($t = 0$), write an integral expression that represents the rabbit population in 2007.

$$100 + \int_0^2 p(t) dt$$

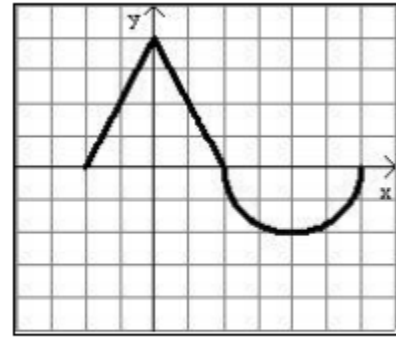
Now for a geometric look at FTC

Do This Problem For Homework

The graph of f' on $-2 \leq x \leq 6$ consists of two line segments and a semicircle as shown at right.

Given that $f(-2) = 5$,

find $f(0)$, $f(2)$, and $f(6)$.



Graph of f'

Here is a nice calculator-friendly FTC problem to ponder

A pizza with a temperature of 95°C is put into a 25°C room when $t = 0$. The pizza's temperature is decreasing at a rate of $r(t) = 6e^{-0.1t}^\circ\text{C}$ per minute. Estimate the pizza's temperature when $t = 5$ minutes.

Another calculator-friendly problem

A particle moving along the x -axis has position $x(t)$ at time t with the velocity of the particle given by $v(t) = 5 \sin(t^2)$. At time $t = 6$, the particle's position is $(4, 0)$. Find the position of the particle when $t = 7$.

And for the first of many water-related problems

Water flows into a tank at a rate of $\frac{dW}{dt} = \frac{1}{75}(600 + 20t - t^2)$ where $\frac{dW}{dt}$ is measured in gallons per hour and t is measured in hours. If there are 150 gallons of water in the tank at time $t = 0$, how many gallons of water are in the tank when $t = 24$?