Free Response Directions

Show all you work. You will be graded on the correctness and completeness of your methods. Answers without supporting work will not receive credit. Be sure to write clearly and legibly and use standard mathematical notation.

A particle is moving along a horizontal line such that its position at time $t,\ t \geq 0,$ is given by the function $s(t) = 2t^3 - 9t^2 + 12t$.

(a) Find the time(s) $t,\ t \geq 0,$ when the particle's velocity is equal to zero. Show all the steps that lead to your solution.

$v(t) = s'(t)$

$v(t) = 6t^2 - 18t + 12$

$0 = 6(t^2 - 3t + 2)$

$0 = 6(t-1)(t-2)$

$v(t) = 0$ at $t = 1$ and $t = 2$

(b) What is the acceleration of the particle at time $t = 2$. Show all the steps that lead to your solution.

$a(t) = v'(t)$

$a(t) = 12t - 18$

$a(2) = 12(2) - 18$

$a(2) = 6$

EXTRA CREDIT [optional]

What is the speed of the particle at time $t = 1.5$? [No need to simplify your numerical solution]

speed at $t = 1.5$

$= \left| 6(1.5)^2 - 18(1.5) + 12 \right|$

Consider the curve given by $xy^2 - x^3y = 6$

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$

$$\frac{d}{dx}xy^2 - \frac{d}{dx}x^3y = \frac{d}{dx}6$$

$$y^2 + 2xy\frac{dy}{dx} - \left[3x^2y + x^3\frac{dy}{dx}\right] = 0$$

$$y^2 + 2xy\frac{dy}{dx} - 3x^2y - x^3\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left[2xy - x^3\right] = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

(b) Find one of the points on the curve whose x-coordinate is 1, and write an equation for the tangent line at this point.

let $x=1$

$$y^2 - y = 6$$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 1$$

Points: $(1,3)$ and $(1,-2)$

$$\left.\frac{dy}{dx}\right|_{(1,3)} = \frac{(3)(1)(3) - (3^2)}{2(1)(3) - 3^3} = 0$$

Hence TAN line at $(1,3)$ is

$$y = 3$$

BONUS

Find the other point on the curve where the x-coordinate is 1, and write an equation for the tangent line at this point. [Should be different than the point in part (b)]

$(1,-2)$ is the other point

$$\left.\frac{dy}{dx}\right|_{(1,-2)} = \frac{-6-4}{-4-1} = 2$$

Hence, TAN line at $(1,-2)$

$$y+2 = 2(x-1)$$