Practice Multiple Choice Exam for First Semester *December 2011*

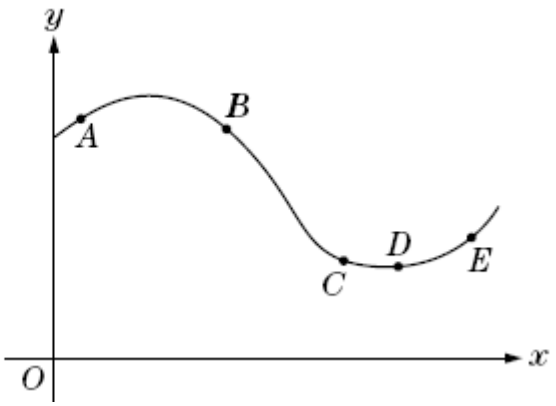
Part I – Non-calculator

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding "bubble" on your Scantron form. Do not spend too much time on any one problem. Please be sure to place your name in the appropriate place on your Scantron form.

1.

At which of the five points on the graph in the figure at the right are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative?

(A) A
(B) B
(C) C
(D) D
(E) E



2.

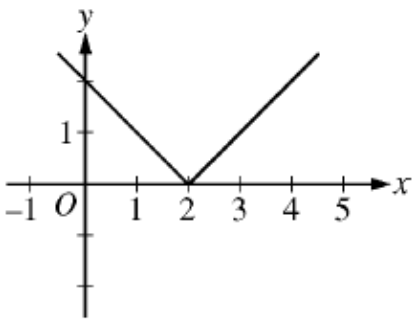
Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

(A) The function has no relative extremum.
(B) The graph of the function has one point of inflection and the function has two relative extrema.
(C) The graph of the function has two points of inflection and the function has one relative extremum.
(D) The graph of the function has two points of inflection and the function has two relative extrema.
(E) The graph of the function has two points of inflection and the function has three relative extrema.

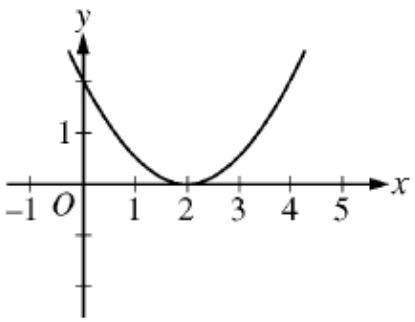
3.

If $f'(x) = |x - 2|$, which of the following could be the graph of $y = f(x)$?

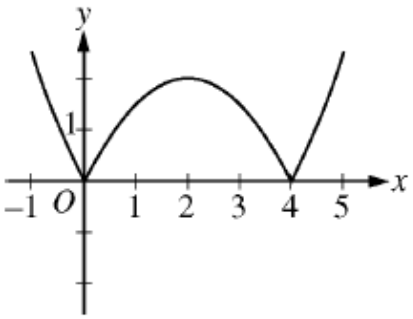
(A)



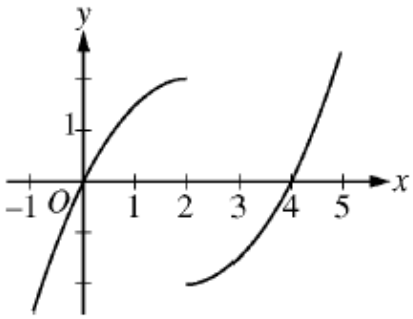
(B)



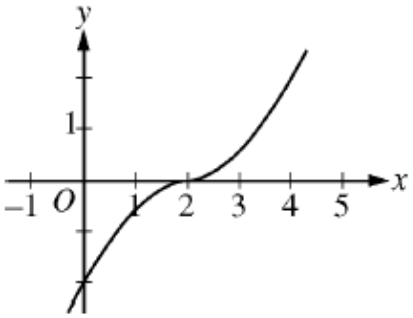
(C)



(D)



(E)



4. Let f be a twice-differentiable function such that its second derivative is given by $f''(x) = x(3x-7)(x+4)^2$. At which value(s) of x does f have point(s) of inflection?

(A) $x = -4$ only

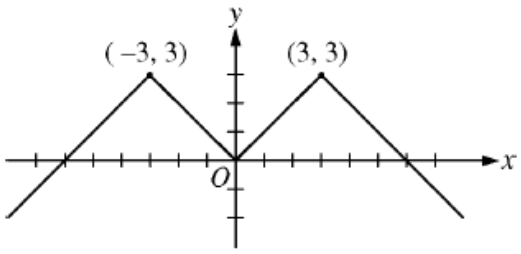
(B) $x = \frac{7}{3}$ only

(C) $x = 0$ only

(D) $x = 0$ and $x = \frac{7}{3}$

(E) $x = 0,\ x = -4,\ x = \frac{7}{3}$

5.



The graph of the even function $y = f(x)$ consists of 4 line segments, as shown above. Which of the following statements about f is false?

(A) $\lim_{x \to 0} (f(x) - f(0)) = 0$

(B) $\lim_{x \to 0} \frac{f(x) - f(0)}{x} = 0$

(C) $\lim_{x \to 0} \frac{f(x) - f(-x)}{2x} = 0$

(D) $\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = 1$

(E) $\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$ does not exist.

6. If $f(x) = \sin^4(3x)$, then $f'(x) =$

(A) $4\sin^3(3x)\cos(3x)$
(B) $12\sin^3(3x)\cos(3x)$
(C) $12\sin(3x)\cos(3x)$
(D) $-12\sin^3(3x)\cos(3x)$
(E) $12\sin^3(3x)$

7. Which of the following must be true about the graph of $f(x) = \sqrt[5]{x^4}$

(A) $f(0)$ does not exist
(B) $\lim_{x \to 0} f(x)$ does not exist
(C) f' is continuous for all $x \in$ Reals
(D) $f'(0) = 0$
(E) $f'(0)$ does not exist

8. What is the average rate of change of the function f given by $f(x) = x^4 - 5x$ on the closed interval [0, 3]?

(A) 8.5
(B) 8.7
(C) 22
(D) 33
(E) 66

9. What is $\lim_{x\to\infty} \dfrac{7-5x+3x^2}{-6x^2+15x+14}$

(A) $\dfrac{-7}{6}$

(B) $\dfrac{-1}{2}$

(C) -2

(D) $\dfrac{1}{2}$

(E) Does not exist

10.
Let f and g be differentiable functions such that

$f(1)=2,$ $f'(1)=3,$ $f'(2)=-4,$

$g(1)=2,$ $g'(1)=-3,$ $g'(2)=5.$

If $h(x)=f\big(g(x)\big),$ then $h'(1)=$

(A) –9 (B) –4 (C) 0 (D) 12 (E) 15

11.
A particle moves along the x-axis so that at any time $t\geq 0$ its position is given by $x(t)=t^3-3t^2-9t+1$. For what values of t is the particle at rest?

(A) No values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3

12.

The $\lim_{h\to 0}\dfrac{\tan 3(x+h)-\tan 3x}{h}$ is

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

13. $\int \sqrt{x}\left(x^2-1\right)dx$

(A) $\left(\frac{2}{3}x^{\frac{3}{2}}\right)\left(\frac{x^3}{3}-1\right)+C$

(B) $\frac{5}{2}x^{\frac{3}{2}}-x^{\frac{1}{2}}+C$

(C) $\frac{2}{7}x^{\frac{7}{2}}-\frac{2}{3}x^{\frac{3}{2}}+C$

(D) $\left(\frac{1}{2}x^{\frac{-1}{2}}\right)(2x)+C$

(E) $\left(\frac{2}{3}x^{\frac{3}{2}}\right)\left(\frac{x^3}{3}-x\right)+C$

14.

Let f be a function that is differentiable on the open interval $(1,10)$. If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros.

II. The graph of f has at least one horizontal tangent.

III. For some c, $2 < c < 5$, $f(c) = 3$.

(A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II, and III

15. $\int dx =$

(A) 0
(B) 1
(C) C
(D) x
(E) $x + C$

16.

If the function f is continuous for all real numbers and if $f(x) = \dfrac{x^2 - 4}{x + 2}$ when $x \neq -2$, then $f(-2) =$

(A) -4 (B) -2 (C) -1 (D) 0 (E) 2

17. Let f be a continuous function on the closed interval $[-3,\ 7]$ and have the properties listed in the table below.

x	$-3<x<-1$	-1	$-1<x<2$	2	$2<x<5$	5	$5<x<7$
$f'(x)$	positive	dne	negative	0	negative	0	positive
$f''(x)$	positive	dne	positive	0	negative	0	positive

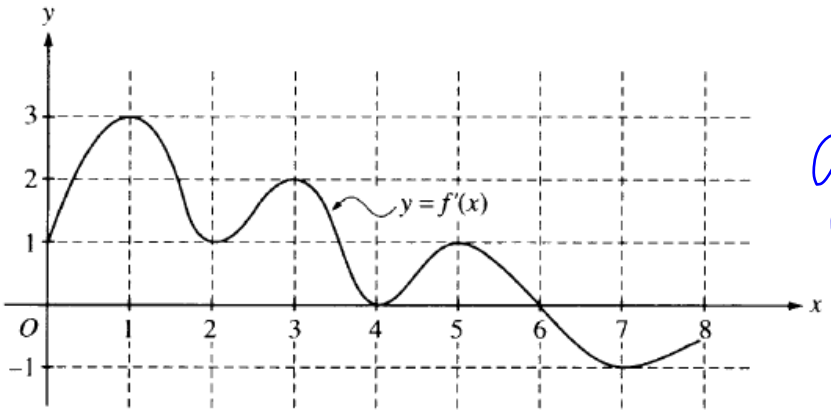
Which of the following statements about f must be true?

(A) f has relative extrema at $x=-1$ and $x=5$ and has points of inflection at $x=-1$ and $x=5$
(B) f has relative extrema at $x=-1$ and $x=2$ and has points of inflection at $x=2$ and $x=5$
(C) f has relative extrema at $x=-1$ and $x=5$ and has points of inflection at $x=-1$ and $x=2$
(D) f has relative extrema at $x=-1$ and $x=5$ and has points of inflection at $x=2$ and $x=5$
(E) f has no relative extrema or points of inflection

18. $\int_{3}^{3} \sqrt{x}\left(x^2-1\right)dx=$

(A) 3
(B) 2
(C) 1
(D) 0
(E) -1

Questions 19 and 20 refer to the graph below



graph of f'

The function f is defined on the closed interval $[0,8]$. The graph of its derivative f' is shown above.

19.

The point $(3,5)$ is on the graph of $y = f(x)$. An equation of the line tangent to the graph of f at $(3,5)$ is

(A) $y = 2$

(B) $y = 5$

(C) $y - 5 = 2(x - 3)$

(D) $y + 5 = 2(x - 3)$

(E) $y + 5 = 2(x + 3)$

20.

How many points of inflection does the graph of f have?

(A) Two
(B) Three
(C) Four
(D) Five
(E) Six

21. The table below gives selected values of the velocity of a particle on the interval [0, 20]

t [time in minutes]	0	5	10	15	20
$v(t)$ [velocity in ft/min]	0	20	50	40	120

Based on the values above, what is the average acceleration of the particle for the interval $0 \le t \le 20$

(A) $120 \ ft/\min^2$ (B) $60 \ ft/\min^2$ (C) $40 \ ft/\min^2$

(D) $6 \ ft/\min^2$ (E) $\frac{1}{6} \frac{ft}{\min^2}$

End of Part I of the Final Exam. Turn in this part of the exam and pick up Part II – the calculator portion of the exam. You may NOT return to this part of the exam once you have turned it in.

My First Semester Practice Multiple-choice December 2011
Part II –-Calculator–Friendly [Are you in radians?!]

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given, and fill in the corresponding "bubble" on your Scantron form. Do not spend too much time on any one problem. Please be sure to place your name in the appropriate place on your Scantron form.

Begin at #28 on your Scantron

28. Let f be the function with first derivative defined by $f'(x) = \sin\left(x^3\right)$ for $0 \le x \le 2$. At what value of x does f attain its maximum value on the closed interval $0 \le x \le 2$?

(A) 0 (B) 1.162 (C) 1.465 (D) 1.845 (E) 2

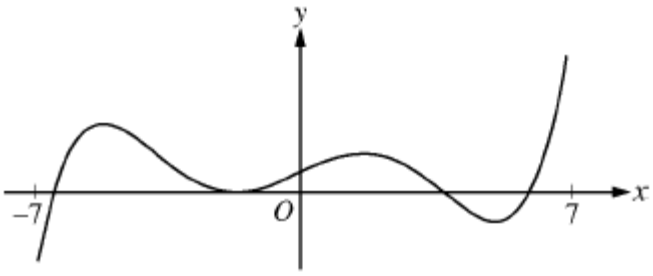
29.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	–2	–3	4

The table above gives values of the differentiable functions f and g and their derivatives at $x = 1$. If $h(x) = (2f(x) + 3)(1 + g(x))$, then $h'(1) =$

(A) –28 (B) –16 (C) 40 (D) 44 (E) 47

30.



Graph of f'

graph of f'

The figure above shows the graph of f', the derivative of the function f, on the open interval $-7 < x < 7$. If f' has four zeros on $-7 < x < 7$, how many relative maxima does f have on $-7 < x < 7$?

(A) One (B) Two (C) Three (D) Four (E) Five

31.

The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

(A) $-\frac{7}{8}$ feet per minute

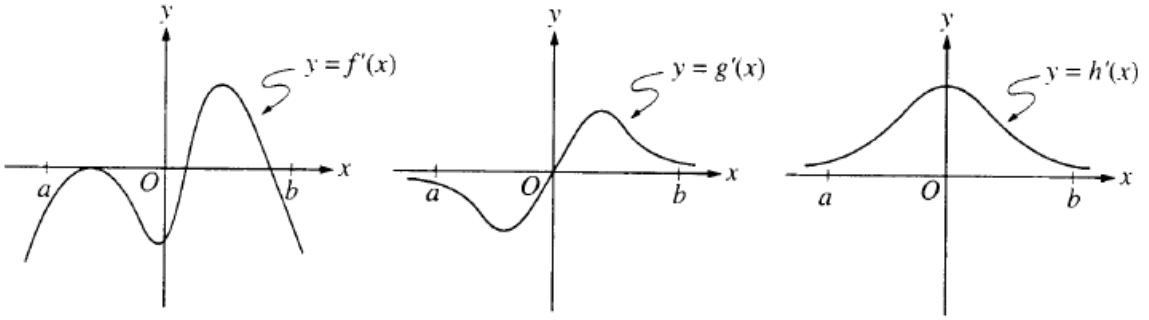
(B) $-\frac{7}{24}$ feet per minute

(C) $\frac{7}{24}$ feet per minute

(D) $\frac{7}{8}$ feet per minute

(E) $\frac{21}{25}$ feet per minute

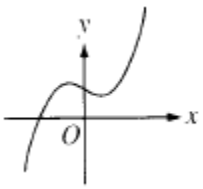
32.



The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

(A) f only
(B) g only
(C) h only
(D) f and g only
(E) f, g, and h

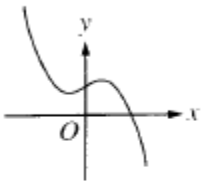
33.



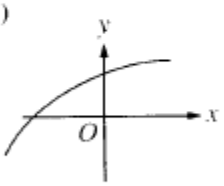
graph of $h(x)$

The graph of $y = h(x)$ is shown above. Which of the following could be the graph of $y = h'(x)$?

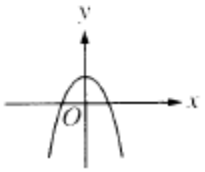
(A)



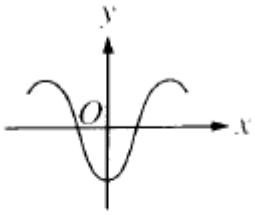
(B)



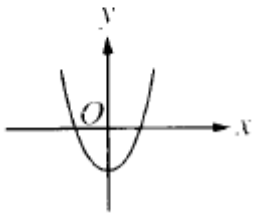
(C)



(D)



(E)



34. $\int_{2}^{2} \frac{1}{x^2+1}\, dx =$

(A) 2π
(B) e
(C) 1
(D) 0
(E) π

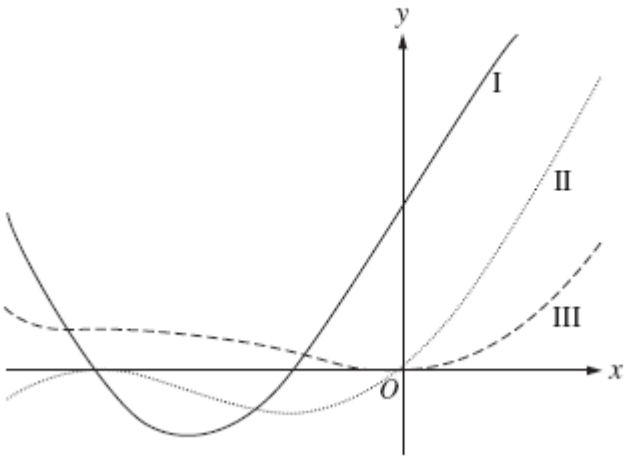
35.

x	0	1	2	3	4	5	6
$f(x)$	0	0.25	0.48	0.68	0.84	0.95	1.00

For the function whose values are given in the table above $\int_{0}^{6} f(x)\, dx$ is approximated by a Riemann Sum using the value of the midpoint of each of three intervals of width 2. The approximation is

(A) 2.64
(B) 3.64
(C) 3.72
(D) 3.76
(E) 4.64

36.



Three graphs labeled I, II, and III are shown above. One is the graph of f, one is the graph of f', and one is the graph of f''. Which of the following correctly identifies each of the three graphs?

	f	f'	f''
(A)	I	II	III
(B)	I	III	II
(C)	II	I	III
(D)	II	III	I
(E)	III	II	I

37. Let f be a function which is continuous on [2, 10] and whose derivative is given by $f'(x) = \dfrac{\cos x}{1+x^2}$. Which of the following are true about $f(x)$ on the interval [2, 10]?

I. $f(x)$ has two relative minima and one relative maximum

II. $f(x)$ has three points of inflection

III. $f(x)$ has one relative minimum and one relative maximum

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

38.

Give a value of c that satisfies the conclusion of the Mean Value Theorem for Derivatives for the function

$$f(x) = -2x^2 + x - 2$$

on the interval [1,3].

a) $\frac{9}{4}$

b) $\frac{3}{2}$

c) $\frac{1}{2}$

d) 2

e) $\frac{5}{4}$