FREE RESPONSE

Show all work in the space provided. All steps must be shown. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as the accuracy of your final answers. Correct answers without **supporting work ["bald" answers]** will NOT receive credit. Justifications require mathematical [non-calculator] reasons. Your work must be expressed in standard mathematical notation. Unless otherwise specified, answers [numeric or algebraic] need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

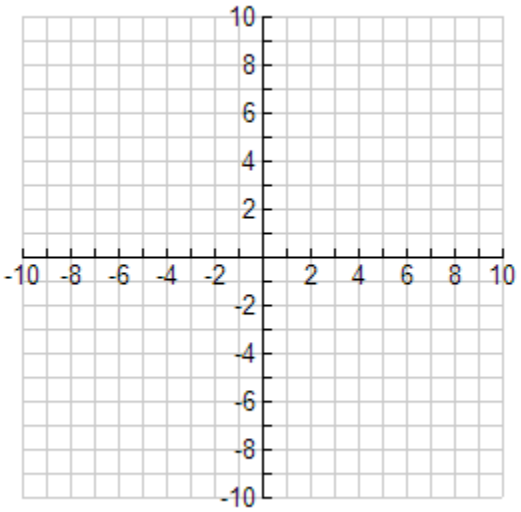
FR1. On the axes provided below, sketch a graph of a function that has all of the following attributes listed below:

I. $\lim_{x \to 3} f(x) = 4$

II. $f(3) = -2$

} REMOVABLE DISC.

III. $\lim_{x \to -3} f(x) = \infty$ VERTICAL ASYMPTOTE



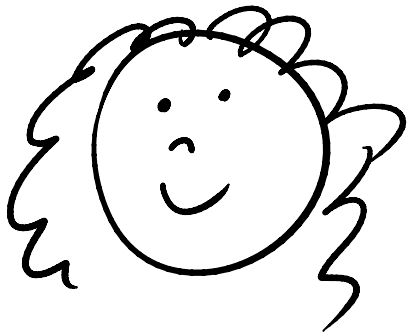
FR2. $f(x)$ and $g(x)$ are continuous functions for all $x \in$ Reals. The table below has values for the functions for selected values of $x$. The function $h(x) = g(f(x)) + 2$.

| $x$ | $f(x)$ | $g(x)$ |
|---|---|---|
| 1 | 3 | 4 |
| 3 | 9 | $-10$ |
| 5 | 7 | 5 |
| 7 | 11 | 25 |

$h(1) = g(f(1)) + 2$
$= g(3) + 2$
$= -10 + 2$
$h(5) = g(f(5)) + 2$
$= g(7) + 2$
$= 25 + 2$

Explain why there must be a value $c$ for $1 < c < 5$ such that $h(c) = 0$.

By IVT there is a c, $1 < c < 5$, such that $h(1) < h(c) < h(5)$ which means that there is a c, $1 < c < 5$, such that $-8 < h(c) < 27$. Hence, there is an $h(c) = 0$ for $1 < c < 5$



FR3. Find $a$ such that the function $f(x)=\begin{cases}\dfrac{4\sin x}{x}, & x<0\\ a+15x, & x\geq 0\end{cases}$ is continuous for all real numbers.

need $f(0)=\lim_{x\to 0} f(x)$

$\lim_{x\to 0^-} f(x)=4$

$\lim_{x\to 0^+} f(x)=a$

Hence, $a=4$

On the axes provided below, sketch a graph of a function that has all of the following attributes listed below:

I. $\lim_{x\to -5^-} f(x) = \infty$

II. $\lim_{x\to -5^+} f(x) = -\infty$

} Vertical Asymptote

III. $\lim_{x\to -2} f(x) = 3$

IV. $f(-2) = 0$

} Remov. Disc

V. $\lim_{x\to 0} f(x) = -\infty$

Vert. Asymptote

VI. $\lim_{x\to 4^-} f(x) = 4$

VII. $f(4) = 1$

VIII. $\lim_{x\to 4^+} f(x) = -4$

} Interesting Jump Disc.