

**My 2010 Chapter 5 AP Calculus AB Test**

**Part I – A CALCULATOR MAY NOT BE USED ON THIS PART OF THE TEST**

*Directions: Solve each of the following problems, using the available space for your scratch work. After examining the form of the choices, decided which is the best of the choices given and fill in the corresponding bubble on your Scantron. No credit will be given for anything in your scratch work. Do not spend too much time on any one problem.*

1.  $\int \tan x \, dx =$

(A)  $\frac{\tan^2 x}{2} + C$

(B)  $\ln|\cot x| + C$

(C)  $\ln|\cos x| + C$

(D)  $-\ln|\cot x| + C$

(E)  $-\ln|\cos x| + C$

2.  $\int_0^2 e^{2x} \, dx =$

(A)  $e^4 - 1$

(B)  $e^4 - 2$

(C)  $2e^4 - 2$

(D)  $\frac{e^4}{2}$

(E)  $\frac{e^4 - 1}{2}$

3.  $\int_1^{e^2} \frac{x^3 + 1}{x} \, dx =$

(A)  $\frac{1}{3}e^6 + \frac{8}{3}$

(B)  $\frac{1}{3}e^6 + \frac{5}{3}$

(C)  $\frac{1}{3}e^6 - \frac{1}{2e^2} + \frac{1}{6}$

(D)  $\frac{1}{3}e^6 - \frac{1}{2e^4} + \frac{1}{6}$

(E)  $\frac{1}{3}e^6 + \frac{7}{3}$

4. If  $f(x) = \cos e^{2x}$ , then  $f'(x) =$

- (A)  $\sin e^{2x}$
- (B)  $2 \sin e^{2x}$
- (C)  $-\sin e^{2x}$
- (D)  $-2 \sin e^{2x}$
- (E)  $-2e^{2x} \sin e^{2x}$

5. If  $f(x) = \begin{cases} \ln 3x, & 0 < x \leq 3 \\ x \ln 3, & 3 < x \leq 4 \end{cases}$  then  $\lim_{x \rightarrow 3} f(x)$  is

- (A)  $\ln 9$
- (B)  $\ln 27$
- (C)  $3 \ln 3$
- (D)  $3 + \ln 3$
- (E) Does not exist

**Bonus Multiple-Choice** [Optional] Mark as # 6 on your Scantron

If  $\int_3^5 f(x-a) dx = 7$  where  $a$  is a constant, then  $\int_{3-a}^{5-a} f(x) dx =$

- (A)  $7 + a$
- (B)  $7 - a$
- (C)  $a - 7$
- (D)  $-7$
- (E)  $7$

**Free Response Directions**

Write all work for each part of each problem in the space provided. Write clearly and legibly. Show ALL your work. You will be graded on the completeness and correctness of your methods as well as the accuracy of your final answers. Correct answers without supporting work [“bald answers”] will not receive any points. Justifications require that you give mathematical [non-calculator] reasons and that you clearly identify functions, graphs, or other objects that you use. Your work must be expressed in standard mathematical notation. Unless otherwise specified, answers need not be simplified. If your answer is given as a decimal approximation, then it should be correct to three decimal places after the decimal point.

Free Response #1

Let  $f$  be the function defined by  $f(x) = k\sqrt{x} - \ln x$  for  $x > 0$ , where  $k$  is a positive constant

- (a) Find  $f'(x)$  and  $f''(x)$
- (b) For what value of the constant  $k$  does  $f$  have a critical point [value] at  $x = 1$ .  
For this value of  $k$ , determine whether  $f$  has a relative minimum, relative maximum, or neither at  $x = 1$ . Justify completely.
- (c) For a certain value of the constant  $k$ , the graph of  $f$  has a point of inflection on the  $x$ -axis. Find this value of  $k$ .

## Free Response #2

A particle moves along the x-axis with position at time  $t$  given by  $x(t) = e^{-t} \sin t$  for  $0 \leq t \leq 2\pi$

- (a) Find the time  $t$  at which the particle is farthest to the left. Justify your answer [with Calculus]

### ***Bonus Free Response [optional]***

Find the value of the constant  $A$  for which  $x(t)$  satisfies the equation

$$Ax''(t) + x'(t) + x(t) = 0 \text{ for } 0 < t < 2\pi$$

***End of Part I. Hand in this portion of the test, KEEP your Scantron, and pick up Part II. You WILL NOT be permitted to go back to this part of the test no matter how much you beg.***

**My Chapter 5 AP Calculus AB Test \_\_\_\_\_**  
**Part II– A GRAPHING CALCULATOR MAY BE NECESSARY FOR SOME**  
**PROBLEM ON THIS PART OF THE TEST**  
**In other words, use your %\$#! Calculator!!!**

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7. If  $f'(x) = 2e^x - 5x^2$ , then for what values of  $x$  the graph of  $f$  have a relative maximum value?
- (A)  $-0.494$
  - (B)  $0.259$
  - (C)  $1.092$
  - (D)  $2.543$
  - (E)  $3.310$
8. Let  $f$  be the function given by  $f(x) = 5e^{3x^3}$ . For what positive value of  $a$  is the slope of the line tangent to the graph of  $f$  at  $(a, f(a))$  equal to 6?
- (A)  $0.142$
  - (B)  $0.344$
  - (C)  $0.393$
  - (D)  $0.595$
  - (E)  $0.714$

**Free Response #3[Same directions as always]**

A particle moves along the y-axis so that its velocity  $v$  at time  $t \geq 0$  is given by

$v(t) = 1 - \tan^{-1}(e^t)$ . At time  $t = 0$ , the particle is at  $y = -1$  [Note:  $\tan^{-1} x = \arctan x$ ]

- (a) Find the acceleration of the particle at time  $t = 2$
- (b) Find the position AND the velocity of the particle at time  $t = 2$ . Is the particle moving toward the origin or away from the origin at time  $t = 2$ . [Justify completely]

***Bonus Free Response [optional]***

Is the speed of the particle increasing or decreasing at time  $t = 2$ ? [Justify completely]