More integration! See page 250

No need to write down, these are in your textbook.

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

The same rules apply for trigonometric integrals – constant multiple rules, sum or difference rule

$$\int 13 \sec^2 \theta \, d\theta$$

$$= 13 \tan \theta + C$$

$$\int (3\theta^2 + \cos \theta) \, d\theta$$

$$= \theta^3 + \sin \theta + C$$

Knowing your trig really helps!

$\int \sin\theta(3+\csc\theta)\, d\theta$ No product rule for integrals

$$= \int (3\sin\theta + 1)\, d\theta$$

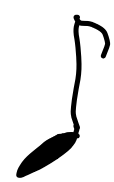
$$= -3\cos\theta + \theta + C$$

$\int \csc\theta(\csc\theta - \cot\theta)\, d\theta$

$$= \int (\csc^2\theta - \csc\theta\cot\theta)\, d\theta$$

$$= -\cot\theta + \csc\theta + C$$

Here's a good one!



$\int \frac{\sin\theta}{1-\sin^2\theta}\, d\theta$

$$= \int \frac{\sin\theta}{\cos^2\theta}\, d\theta$$

$$= \int \sec\theta\tan\theta\, d\theta$$

$$= \sec\theta + C$$

$\frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$

$= \tan\theta\sec\theta$

STORM SAYS!

More "diff EQ"

Let $\frac{dy}{dx} = 4x$ where my initial condition is that $f(0) = 6$

the derivative is 4x and has the point (0,6)

$$\frac{dy}{dx} = 4x$$

$$dy = 4x\,dx$$

SEPARATE

$$\int dy = \int 4x\,dx$$

INTEGRATE both SIDES

$$y + C_1 = 2x^2 + C_2$$

Combine the constants into one constant, call it C and since we are solving for a y, place the constant on the side with x

$$y = 2x^2 + C$$

SOLVE FOR C

Now use our initial condition to solve the particular solution [rather than a general solution]

To find C use $f(0) = 6$ in our general solution

$$y = 2x^2 + C$$

$$6 = 0 + C$$

$$6 = C$$

solve for y

Hence, $f(x) = 2x^2 + 6$

This is the particular solution that fulfills all of the given information.

is $f'(x) = 4x$ ☺

is $f(0) = 6$ ☺

Let's try this one:

$\frac{dy}{dx} = 6x^2$ With our initial condition $f(0) = -1$

$$dy = 6x^2\,dx$$ SEPARATE

$$\int dy = \int 6x^2\,dx$$ INTEGRATE

$$y = 2x^3 + C$$ SOLVE FOR C

$$-1 = 0 + C$$

$$y = 2x^3 - 1$$ SOLVE FOR y

Now let's see how we can use this in a physics setting.
[Physics students – please use Calculus for this class]
See page 257 #67

A ball is thrown vertically upward from a height of 6 feet with an initial velocity of 60 feet per second. How high will the ball go?

Given: $a(t) = -32\ \dfrac{ft}{\sec^2}$, $v(0) = 60\ \dfrac{ft}{\sec}$, $s(0) = 6$ feet

We know that $a(t) = v'(t)$ so $v(t) = \int a(t)\,dt$

$$v(t) = \int -32\ dt$$
$$v(t) = -32t + C$$

Now we need to use our initial conditions to find C

Since $v(0) = 60$

$$60 = 0 + C$$

So, $v(t) = -32t + 60$

We are still looking for the maximum height so we will eventually need a position function.

Since $v(t) = s'(t)$, then $s(t) = \int v(t)\ dt$

$$s(t) = \int(-32t + 60)\,dt$$
$$s(t) = -16t^2 + 60t + C$$

$s(t) = -16t^2 + 60t + C$

Once again, use our initial condition to find C.

Since $s(0) = 6$, then $6 = -16(0^2) + 60(0) + C$
Hence, $C = 6$
So, $s(t) = -16t^2 + 60t + 6$

Now to answer the question. To find maximum height we first need to let $v(t) = 0$

$0 = -32t + 60$
$t = 1.875$ sec

ENDpoints are NOT CANDIDATES

at $t = 1.875$ $v(t)$ changes from + to −

Now find the maximum height

find $s(1.875)$

$s(1.875) = 62.25$ ft
max height

♪ #68 gives us a general equation for position [in feet]

$$s(t) = -16t^2 + (v_0)t + (s_0)$$
Where $v_0 = v(0) =$ initial velocity
And $s_0 = s(0) =$ initial position

Try #73 on page 257 [Since we are in meters, then $a(t) = -9.8\frac{m}{\sec^2}$ [show all integration steps]

Is there a general equation for meters?

$s(0) = 2m \qquad v(0) = 10\ \frac{m}{sec}$

$v(t) = \int a(t)\,dt$

$= \int -9.8\,dt$

$v(t) = -9.8t + C$

use $v(0) = 10\ \frac{m}{sec}$

$10 = 0 + C$

$v(t) = -9.8t + 10$

$s(t) = \int v(t)\,dt$

$= \int (-9.8t + 10)\,dt$

$s(t) = -4.9t^2 + 10t + C$

use $s(0) = 2\text{m}$

$2 = 0 + 0 + C$

$s(t) = -4.9t^2 + 10t + 2$

To find when max height occurs

Let $v(t) = 0$

$0 = -9.8t + 10$

$1.020 = t$

at $t = 1.020$ $v(t)$ changes

from + to −

To find max height find $s(1.020)$

$s(1.020) = 7.102\text{ m}$

max height

Homework: page 255 #36, 38, 40, 41, 42

If you have forgotten your trigonometry, then look it up