Fundamental Theorem of Calculus [FTC]

If f is continuous on $[a,b]$ AND F is an anti-derivative of f on $[a,b]$, then

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

Easier Version:

$$\boxed{\int_a^b f'(x)\,dx = f(b) - f(a)}$$

Notice that there is no "+ C"
Here is why:

$$\int_{11}^{15} f'(x)\,dx$$

$$= f(x) + C \Big|_{11}^{15}$$

$$= [f(15) + C] - [f(11) + C]$$

$$= f(15) - f(11)$$

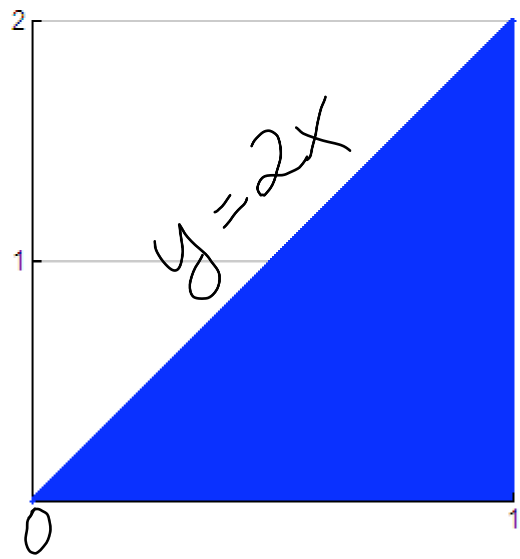
Example:

$$\int_0^1 2x\,dx$$

$$= x^2 \Big|_0^1$$

$$= 1^2 - 0^2 = 1$$

Does this agree with the geometric way to find a definite integral?



Area of shaded region: $\left(\frac{1}{2}\right)(1)(2) = 1$

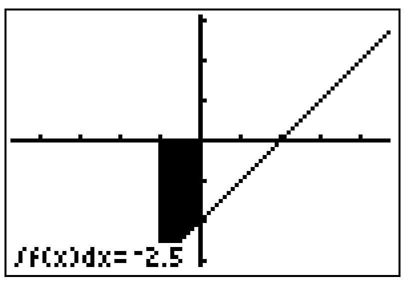
$$\int_{-1}^{0}(x-2)\,dx$$

$$= \frac{1}{2}x^2 - 2x \Big|_{-1}^{0}$$

$$= (0-0) - \left(\frac{1}{2}+2\right)$$

$$= -2.5$$

Here's what it looks like:



So we are now back to using anti-derivatives rather than summing up the area of rectangles.

$\int_0^1 (2t-1)^2\, dt$ We need to simplify the integrand!

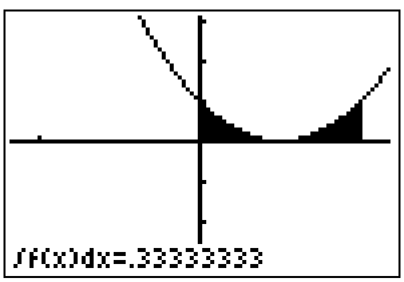
$= \int_0^1 \left(4t^2 - 4t + 1\right) dt$ Now find the anti-derivative

$= \dfrac{4t^3}{3} - 2t^2 + t \Big|_0^1$

$= \left(\dfrac{4}{3} - 2 + 1\right) - \left(0 - 0 + 0\right)$

$= \dfrac{1}{3}$

Let's validate our answer by checking with "Billy Bob"



MATH #9

fnInt(y1, x, LB, UB)

All the previously-stated rules of integration apply. Let's use them to find the following.

$\int_1^2 \left(\dfrac{3}{x^2} - 1\right) dx$

$= \int_1^2 \left(3x^{-2} - 1\right) dx$

$= \dfrac{-3}{x} - x \Big|_1^2$

fnInt(y1, x, 1, 2)

$$= \left(-\frac{3}{2}-2\right)-(-3-1)$$

$$= \frac{1}{2}$$

$$\int_1^4 \frac{u-2}{\sqrt{u}}\,du$$

$$= \int_1^4 \left(u^{\frac{1}{2}} - 2u^{-\frac{1}{2}}\right) du$$

$$= \frac{2}{3}u^{\frac{3}{2}} - 4u^{\frac{1}{2}} \Big|_1^4$$

$$= \left[\frac{2}{3}\left(4^{\frac{3}{2}}\right) - 4\left(4^{\frac{1}{2}}\right)\right] - \left[\frac{2}{3} - 4\right]$$

$$= \left[\frac{16}{3} - \frac{24}{3}\right] - \left[-\frac{10}{3}\right]$$

$$= \frac{2}{3}$$

$$\int_0^{\pi} (1+\sin x)\,dx$$

$$= x - \cos x \Big|_0^{\pi}$$

$$= (\pi - \cos\pi) - (0 - \cos 0)$$

$$= (\pi + 1) - (-1)$$

$$= \pi + 2$$

$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sec^2 x\,dx$$

$$= \tan x \Big|_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$$

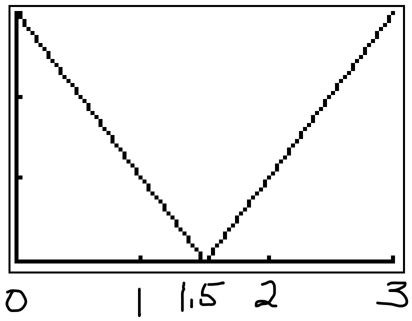
$$= \tan\left(\frac{\pi}{6}\right) - \tan\left(-\frac{\pi}{6}\right)$$

$$= \frac{1}{\sqrt{3}} - \left(-\frac{1}{\sqrt{3}}\right)$$

$$= \frac{2}{\sqrt{3}}$$

And now for something slightly harder:

$$\int_0^3 |2x-3| \, dx$$



We can re-write this as a piece-wise function:

$$f(x) = \begin{cases} -(2x-3), & 0 \le x \le 1.5 \\ 2x-3, & 1.5 < x \le 3 \end{cases}$$

We can now re-write our integral to ensure that we get the correct answer:

$$\int_0^3 |2x-3| \, dx = \int_0^{1.5} -(2x-3)\,dx + \int_{1.5}^3 (2x-3)\,dx$$

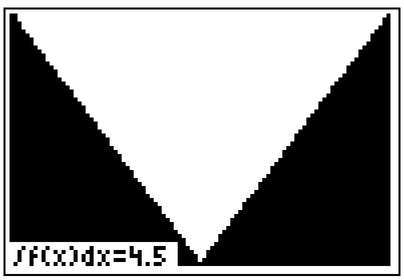
$$= \left[-x^2 + 3x \Big|_0^{1.5} \right] + \left[x^2 - 3x \Big|_{1.5}^3 \right]$$

$$= \left[\left(-(1.5^2) + 3(1.5)\right) - 0 \right] + \left[(3^2 - 3^2) - (1.5^2 - 3(1.5)) \right]$$

$$= 2.25 \quad + 2.25$$

$$= 4.5$$

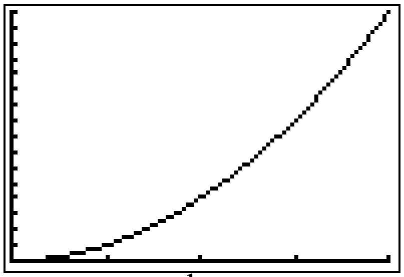
Billy Bob's validation:



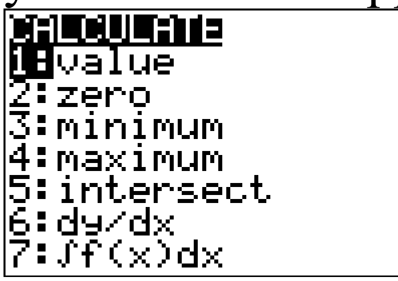
How to find a definite integral with our TI

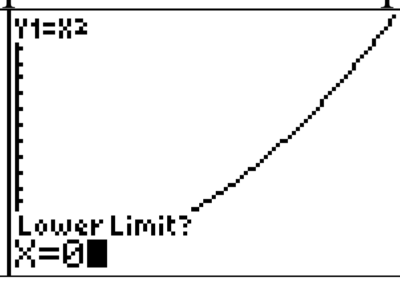
Our example: $\int_0^4 x^2\,dx$

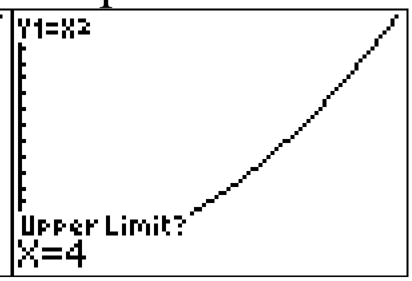
1. Put the function into y_1
2. Enter your lower and upper bounds as your x-min and x-max and "Zoom-Fit"



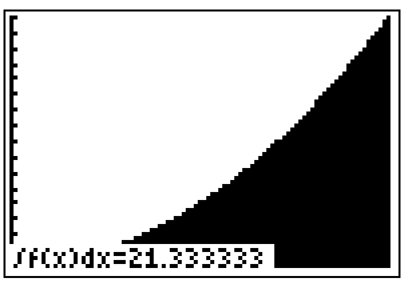
3. Use Calc [2^{nd} Trace] and choose option 7 and enter your lower and upper bounds when prompted.







4. Your graph should be shaded in the proper region with the numerical answer in the lower right corner.



Home Screen
math #9
fnInt(y1, x, LB, UB)

Definite Integrals that we know so far:

$$\int_a^b x^n\,dx = \left.\frac{x^{n+1}}{n+1}\right|_a^b \qquad \text{for } n \neq -1$$

$$\int_a^b \cos x \, dx = \sin b - \sin a$$

$$\int_a^b -\sin x \, dx = \cos b - \cos a$$

$$\int_a^b \sec^2 x \, dx = \tan b - \tan a$$

$$\int_a^b \sec x \tan x \, dx = \sec b - \sec a$$

$$\int_a^b -\csc^2 x \, dx = \cot b - \cot a$$

$$\int_a^b -\csc x \cot x \, dx = \csc b - \csc a$$

Homework: page 291 #6, 7, 10, 14, 20, 28, 30, 32 *show all steps* but feel free to validate your answers with Billy Bob