A Review of FTC so far

$\int f'(x)\,dx$

$= f(x) + C$

$\int_1^7 f'(x)\,dx$

$= f(x)\Big|_1^7$

$= f(7) - f(1)$

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$\int 2x\,dx$

$= x^2 + C$

$\int_1^3 2x\,dx$

$= x^2\Big|_1^3$

$= 3^2 - 1^2$

****

$\int \cos x\,dx$

$= \sin x + C$

$\int_0^{\frac{\pi}{2}} \cos x\,dx$

$= \sin x\Big|_0^{\frac{\pi}{2}}$

$= \sin\frac{\pi}{2} - \sin 0$

$= 1$

Second Fundamental Theorem of Calculus

$$\int_0^x \cos t \, dt$$

♪ Please look at the variables

$$= \sin t \Big|_0^x$$

$$= \sin x - \sin 0$$

$$= \sin x$$

Now consider the following:

Order of Operations – do the integration first!

$$\frac{d}{dx} \int_0^x \cos t \, dt$$

$$= \frac{d}{dx}\left[\sin x\right]$$

$$= \cos x$$

Hmm! Interesting!

Let's look at a different problem.:

$$\int_0^x 2t\,dt =$$

$$= t^2 \Big|_0^x$$

$$= x^2 - 0^2$$

$$= x^2$$

Now consider this:

$$\frac{d}{dx}\int_0^x 2t\,dt$$

$$= \frac{d}{dx}\left[x^2\right]$$

$$= 2x$$

Very interesting! Is there a pattern?
Let's try one more to see.

$$\int_0^x (2t-5)\,dt$$

$$= t^2 - 5t \Big|_0^x$$

$$= (x^2 - 5x) - (0 - 0)$$

$$= x^2 - 5x$$

Now find

$$\frac{d}{dx}\int_0^x (2t-5)\,dt$$

$$= \frac{d}{dx}\left[x^2 - 5x\right]$$

$$= 2x - 5$$

What seems to be happening?

Second FTC

If f is continuous on an open interval, I, containing a, then for every x in the interval

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

$a \in$ Reals

♪ $\int_a^x f(t)\,dt$ is sometimes called an *accumulation function* because we are accumulating area or accumulating rate of change. Also ♪ the upper bound must be x. If it is not x, [or t] then we will need to do some more work! [or panic]

Let's try a few:

$$\frac{d}{dx}\int_1^x \sqrt[3]{t}\,dt$$

$$= \sqrt[3]{x}$$

♪ $a = 1$ not zero

Does this verify the Second FTC?

Can we use a shortcut?

$$\frac{d}{dx}\int_3^x t^5 \, dt$$

$$= x^5$$

Beware of the following:

$$\frac{d}{dx}\int_x^3 t^5 \, dt$$

$$= \frac{d}{dx}\left[-\int_3^x t^5 \, dt\right]$$

$$= -x^5$$

What should we do?!

Re-write

Let's try:

$$\frac{d}{dx}\int_{2}^{x}\left(\sin t+t^{2}\right)dt$$

$$= \sin x + x^2$$

$$\frac{d}{dx}\int_{x}^{2}\left(\sin t+t^{2}\right)dt$$

$$= \frac{d}{dx}\left[-\int_{2}^{x}(\sin t + t^2)\,dt\right]$$

$$= -\left[\sin x + x^2\right]$$

$$\frac{d}{dx}\int_{10}^{x}\left(t^{3}-2t^{2}+t\right)dt$$

$$= x^3 - 2x^2 + x$$

$$F(x) = \int_{\frac{\pi}{2}}^{x^2} \cos t \, dt \qquad \text{Find } F'(x)$$

NOTE: We can't use a shortcut! Oh no!

$$F'(x) = \frac{d}{dx}\int_{\frac{\pi}{2}}^{x^2} \cos t \, dt$$

$$= \frac{d}{dx}\left[\sin t \Big|_{\frac{\pi}{2}}^{x^2} \right]$$

$$= \frac{d}{dx}\left[\sin x^2 - \sin \frac{\pi}{2} \right]$$

$$= \frac{d}{dx}\left[\sin x^2 - 1 \right]$$

$$= 2x \cos(x^2)$$

Isn't this the same as:

$$\frac{d}{dx}\int_{\frac{\pi}{2}}^{x^2} \cos t \, dt$$

$$= \cos\left(x^2\right) \bullet \frac{d}{dx}\left(x^2\right)$$

$$= 2x\cos\left(x^2\right)$$

Another example:

Let $F(x) = \int_3^{3x} 2t\,dt$ Find $F'(x)$

$$F'(x) = \frac{d}{dx}\int_3^{3x} 2t\,dt$$

$$= \frac{d}{dx}\left[t^2 \Big|_3^{3x}\right]$$

$$= \frac{d}{dx}\left[9x^2 - 9\right]$$

$$= 18x$$

Once again, $F'(x) = 2(3x) \bullet \frac{d}{dx}(3x)$

Can you think of a shortcut?

REPLACE t WITH UPPER BOUND AND then MULTIPLY by DERIVATIVE OF UPPER BOUND

Find $F'(x)$ for the given:

$$F(x) = \int_5^{x^3} \sec^2 t\,dt$$

$$F'(x) = \sec^2(x^3) \cdot \frac{d}{dx}x^3$$

$$= 3x^2 \sec^2(x^3)$$

$$F(x) = \int_1^{\cos x} 2t\,dt$$

$$F'(x) = 2\cos x \bullet \frac{d}{dx}\cos x$$

$$= -2\cos x \sin x$$

Now for the weirdest! [There are two different ways]

Let $F(x)=\int_{x}^{x^2}\cos t\,dt$ Find $F'(x)$

$$F'(x)=\frac{d}{dx}\int_{x}^{x^2}\cos t\,dt$$

$$=\frac{d}{dx}\left[\int_{x}^{0}\cos t\,dt+\int_{0}^{x^2}\cos t\,dt\right]$$

$$=\frac{d}{dx}\left[-\int_{0}^{x}\cos t\,dt+\int_{0}^{x^2}\cos t\,dt\right]$$

$$=\left(-\cos x+2x\cos x^2\right)$$

The alternate way of doing this:

Let $F(x)=\int_{x}^{x^2}\cos t\,dt$ Find $F'(x)$

$$F'(x)=\frac{d}{dx}\left[\int_{x}^{x^2}\cos t\,dt\right]$$

$$=\frac{d}{dx}\left[\sin t\Big|_{x}^{x^2}\right]$$

$$=\frac{d}{dx}\left[\sin x^2-\sin x\right]$$

$$=2x\cos x^2-\cos x$$

Try the following using a shortcut if it is applicable:
Find $F'(x)$ for the following:

$$F(x) = \int_1^x \frac{t^3}{t^4+1}\, dt$$

yay STRAWBERRY SHORT CUT

$$F'(x) = \frac{x^3}{x^4+1}$$

$$F(x) = \int_1^{x^2} \frac{t^3}{t^4+1}\, dt$$

$$F'(x) = \frac{(x^2)^3}{(x^2)^4+1} \cdot \frac{d}{dx}(x^2)$$

$$= \frac{2x}{x^8+1}$$

$$F(x) = \int_1^{\sin x} \sqrt[3]{t}\, dt$$

$$F'(x) = \sqrt[3]{\sin x}\left(\frac{d}{dx}\sin x\right)$$

RASPBERRY SHORTCUT

$$= \sqrt[3]{\sin x}\,(\cos x)$$

$$F(x) = \int_0^{x^3} \sin\left(t^3\right) dt$$

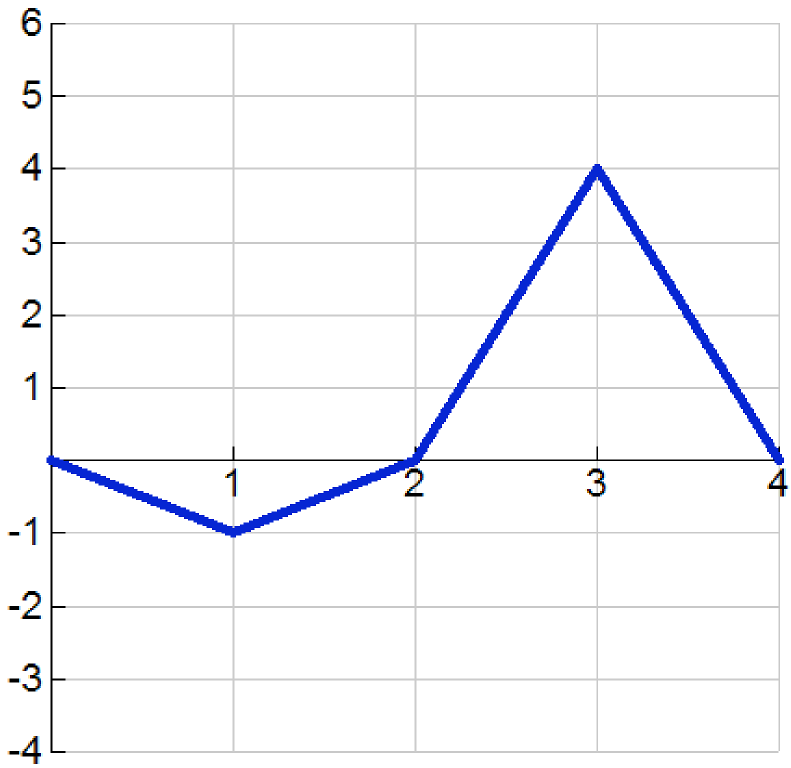
$$F'(x) = \sin\,(x^3)^3 \; \frac{d}{dx}\, x^3$$

$$= 3x^2 \sin(x^9)$$

How to use the Accumulation Function:
This is an accumulation function

$$g(x)=\int_0^x f(t)\,dt$$

Given $g(x)=\int_0^x f(t)\,dt$ and the graph below



$g(x)=\int_0^x f(t)\,dt$

graph of f

Hey! this is also the graph of $g'(x)$

Find the following:
$g(0)$, $g(1)$, $g(2)$, $g(4)$

$$g(x) = \int_0^x f(t)\,dt$$

$$g(0) = \int_0^0 f(t)\,dt$$

$$g(0) = 0$$

$$g(1) = \int_0^1 f(t)\,dt$$
$$= -\tfrac{1}{2}(1)(1)$$
$$g(1) = -\tfrac{1}{2}$$

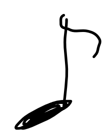
$$g(2) = \int_0^2 f(t)\,dt$$
$$= -\tfrac{1}{2}(2)(1)$$
$$= -1$$

$$g(4) = \int_0^4 f(t)\,dt$$
$$= \int_0^2 f(t)\,dt + \int_2^4 f(t)\,dt$$
$$= -1 + 4$$
$$= 3$$

Find $g'(x)$

$$g'(x) = \frac{d}{dx}\int_0^x f(t)\,dt$$

$$g'(x) = f(x)$$



YOU MUST ALWAYS SHOW THIS WORK

Find $g'(1)$, $g'(3)$

$$g'(1) = f(1)$$
$$g'(1) = -1$$

$$g'(3) = f(3)$$
$$g'(3) = 4$$

Find the equation of the tangent line to the graph of g at $x=1$

$g(1) = -\frac{1}{2}$ $\qquad$ $g'(1) = -1$

Point $\qquad$ SLOPE

$y + \frac{1}{2} = -(x-1)$

Homework:
Page 293 #81-88 all
Please use standard mathematical notation and show your work like the examples in class