Introduction to Integration

Whose derivative is $5x^4$?

If $f'(x) = 5x^4$, then $f(x) = x^5 + C$, where $C \in$ real numbers.

Because: If $f(x) = x^5$, then $f'(x) = 5x^4$

If $f(x) = x^5 + 3$, then $f'(x) = 5x^4$

If $f(x) = x^5 - 4$, then $f'(x) = 5x^4$

Definition of an *anti-derivative*:

A function F is an anti-derivative of f on an interval I if

$F'(x) = f(x)$ for all x in I.

Consider $f(x) = 2x$

Let $F(x)$ be an anti-derivative of $f(x)$.

Then $F(x) = x^2 + C,\ C \in$ real numbers

Think of $F(x)$ as the family of functions whose derivative is equal to $2x$

If $f(x) = 2x$ and $f(x)$ is a derivative of $F(x)$, then we can say that $\dfrac{dy}{dx} = 2x$

This is called a *differential equation.*

Our first differential equation:

$$\frac{dy}{dx} = 5$$

$C \in \mathbb{R}$

Then $\quad y = 5x + C$

[Note: Ch. 6 is all about "diff eq"]

New notation!!!!

$$\frac{dy}{dx} = f(x)$$

$$dy = f(x)\ dx$$

$$y = \int f(x)\ dx$$

$$y = F(x) + C$$

$\int$

$\int$ is an integral sign

$\int f(x)\, dx$ is called an indefinite integral.

♪ We will also learn definite integrals.

Indefinite integral is a synonym for anti-derivative. Finding the anti-derivative is called anti-differentiating or is called integrating.

In general, for $C \in$ real numbers

$$\int F'(x)\,dx = F(x) + C$$

OR

$$\int f(x)\,dx = F(x) + C$$ where $F'(x) = f(x)$

OR

$$\int f'(x)\,dx = f(x) + C$$

$\int s'(t)\,dt = s(t) + C$

♪ The solution to our indefinite integral is a family of functions. If you forget your "+ C" , then you are a "chucklehead"! Don't be a chucklehead! Remember, FHS Calc AB – we're smart and we're pretty! NO chuckleheads here!

See page 250 for Rules of Integration [They are similar to our Rules of Differentiation.]

Derivatives	*Indefinite Integrals*
$\frac{d}{dx}[c] = 0$ $c \in$ Reals	$\int 0\,dx = c$

Derivatives	*Indefinite Integrals*
$\frac{d}{dx}[kx] = k$	$\int k\ dx = kx + C$
$k \in$ Reals	
$\frac{d}{dx} 5x = 5$	$\int 5 dx = 5x + C$
$\frac{d}{dx}[k\ f(x)] = k\ f'(x)$	$\int k\ f'(x)\, dx = k\ f(x) + C$
$\frac{d}{dx}[5 f(x)] = 5 f'(x)$	$\int 5 f'(x) dx = 5 f(x) + C$
$\frac{d}{dx}[f(x) \pm g(x)]$	$\int [f'(x) \pm g'(x)]\, dx$
$= f'(x) \pm g'(x)$	$= f(x) \pm g(x) + C$
$\frac{d}{dx} x^n = n x^{n-1}$	$\int n x^{n-1}\, dx = x^n + C$
$n \neq 0$	$\int 5x^4 dx = x^5 + C$
Power Rule	
$\frac{d}{dx} x^5 = 5x^4$	

Now in action! [How can we check our answer?]

$$\int -2\,dx = -2x + C$$

$\frac{d}{dx}(-2x+C) = -2$

$$\int 17\,dx = 17x + C$$

$$\int 5\,f'(x)\,dx = 5f(x) + C$$

$$\int 3x^2\,dx = x^3 + C$$

$\frac{d}{dx}(x^3+C) = 3x^2$

$$\int \left(3x^2 + 17\right)dx = x^3 + 17x + C$$

$$\int 4x^3\,dx = x^4 + C$$

$\frac{d}{dx}(x^4+C) = 4x^3$

In general, $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$

$$\int x^7 \, dx = \frac{x^8}{8} + C$$

$$\frac{d}{dx}\left(\frac{1}{8}x^8 + C\right) = x^7$$

$$\int x^{-3} \, dx = \frac{x^{-2}}{-2} + C$$

Slightly harder:

$\int 7x^2 \, dx$

We can think of this as:

$$7\int x^2 \, dx = 7\left[\frac{x^3}{3} + C\right]$$

CONSTANT MULTIPLE RULE

$$= \frac{7x^3}{3} + C$$

Remember, C is a non-specified constant

Some handy tricks:

Rewriting as x^n

Simplifying if it helps

$\int \sqrt[5]{x}\ dx$ Rewrite

$$= \int x^{\frac{1}{5}} dx = \frac{5}{6} x^{\frac{6}{5}} + C$$

$\int \frac{1}{x^5}\ dx$ Rewrite

$$= \int x^{-5} dx$$

$$= -\frac{1}{4} x^{-4} + C$$

$\int \left(\frac{1}{7x^3}\right) dx$ Rewrite

$$= \frac{1}{7} \int x^{-3} dx$$

$$= \frac{1}{7}\left(\frac{x^{-2}}{-2}\right) + C$$

$$= -\frac{1}{14x^2} + C$$

Note: Do not fall in love with the integral sign!

Homework: No need to check, just write the integral and find the indefinite integral

Page 255 #16, 18, 22, 28, 32, 34
Please use standard mathematical notation and do not forget “+C”
[which includes using vertical format]