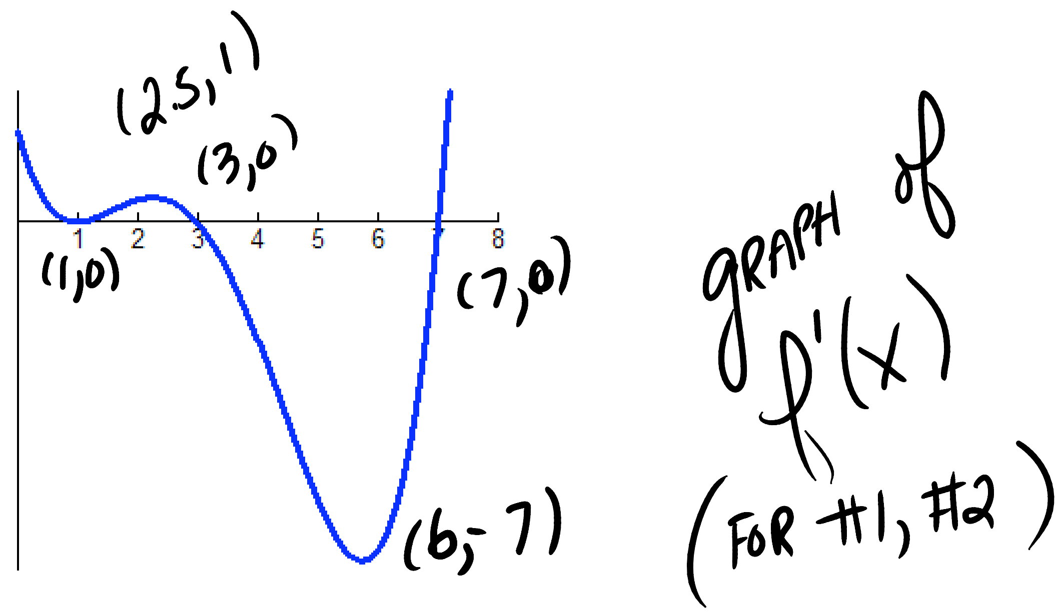
My Chapter 3 Take-Home Project ________________________________

I, __________________________________, claim that all work on this project is my own and that I have received NO help from anyone living, dead, or undead.

[You may consult your own notes and your textbook but may NOT consult with anyone. This includes asking someone, including Ms. McCleary, on how to do a problem.]



1. The graph of the first derivative of a function is shown above. At what values of x does the function, $f(x)$, have relative extrema? Justify completely.

At $x=3$ the graph of f' changes from positive to negative values. Hence the graph of f has a relative maximum at $x=3$.

At $x=7$ the graph of f' changes from negative to positive values. Hence the graph of f has a relative minimum at $x=7$

Common errors:

Giving the points (3, 0) and/or (7, 0). These are points on the graph of the first derivative and not necessarily on the graph of the function *f.*

2. At what values of x does the function $f(x)$, have points of inflection? Justify completely.

At $x=1$ and $x=6$ the graph of f' changes from decreasing to increasing.

At $x=2.5$ the graph of f' changes from increasing to decreasing.

Hence, the graph of f has points of inflection at $x=1,\ 2.5,\ and\ 6$

Common errors:

Talking about values of the second derivative without any proof. Since you are given the graph of the first derivative, then you must address the graph of the first derivative. This was covered in class.

Giving ordered pairs in your answer. We have no ordered pairs for the function f.

3. Let $f(x)$ be the function defined by $f(x)=k+12x+3x^2-2x^3$, where k is a constant.

(a) On what interval(s) is the function increasing? Justify your answer.

$f'(x)=12+6x-6x^2$

$0=12+6x-6x^2$ at $x=-1$ and $x=2$

$f'(x)>0$ on $(-1,2)$ hence $f(x)$ is increasing on $[-1,2]$

I accepted (-1, 2) for the solution this time, but will not on the final exam.

(b) If the relative maximum value of f is 30, then what is the value of k? [Justify]

At $x=2$ $f'(x)$ changes from positive to negative values. Hence f has a relative maximum value at $x=2$.

Then $f'(2)=0$ and $f(2)=30$

Then $30=k+12(2)+3(4)-2(8)$

Hence $k=10$

(c) Find the interval where the function f is concave up. [Justify]

$f''(x) = 6 - 12x$

Let $0 = 6 - 12x$

Then $x = 2$

$f''(x) > 0$ on $\left(-\infty, \frac{1}{2}\right)$ Hence, f is concave up on $(-\infty, \frac{1}{2}]$

Be sure to write the intervals correctly.

4. Let f be a twice-differentiable function such that $f(10) = 25$ and $f(25) = 10$. And, let $g(x)$ be the function given by $g(x) = f(f(x))$.

(a) Explain why [using Calculus] there must be value c, $10 < c < 25$, such that $g'(c) = 1$.

$$g(10) = f(f(10)) = f(25) = 10 \qquad g(25) = f(f(25)) = f(10) = 25$$

You must clearly show this

By the Mean Value Theorem, there is a $c, 10 < c < 25,$ such that $g'(c) = \frac{g(25) - g(10)}{25 - 10}$.

Since $\frac{g(25) - g(10)}{25 - 10} = \frac{25 - 10}{25 - 10} = 1$, then $g'(c) = 1$ for $10 < c < 25$

Please be careful with how you phrase your justification. If I wrote "wording" on your project, then you need to learn how to justify in a logical manner.

(b) Show that $g'(10) = g'(25)$. Use this result to show that there must be a value $k, 10 < k < 25$, such that $g''(k) = 0$.

$g'(x) = f'(f(x)) \bullet f'(x)$ or $g'(x) = [f'(f(x))][f'(x)]$

So, $g'(10) = f'(f(10))\, f'(10) = f'(25)\, f'(10)$ AND $g'(25) = f'(f(25))\, f'(25) = f'(10)\, f'(25)$

By the Mean Value Theorem there is a $k, 10 < k < 25$, such that $g''(k) = \frac{g'(25) - g'(10)}{25 - 10}$. Since $\frac{g'(25) - g'(10)}{25 - 10} = 0$ then there must exist a $k, 10 < k < 25$ such that $g''(k) = 0$

The table below gives values of the velocity, $v(t)$, **of a Inferi [An "inferi" is** sort of like a zombie but made by the Dark Lord] at selected times.

t *(sec)*	0	1	3	6	10	15
$v(t)$ *m/sec*	5	1	-1	5	10	13

(a) Is there a time during $0 < t < 15$, that the velocity is equal to 9 *m/sec*

Justify completely.

By the Intermediate Value Theorem, there is a $t, 0 < t < 15$, such that $v(0) < v(t) < v(15)$. Since $v(0) = 5$ and $v(15) = 13$, then there must exist a $v(t) = 9$ on $0 < t < 15$

A similar argument can be made on other subintervals but you needed to reflect that in your justification.

The number of people who are still using other functions such as f(x) in their justifications is astounding. Notation makes a difference.

(b) Find an approximation for the acceleration at time $t = 2$ and indicate units. [Show all work]

$a(t) = v'(t)$

$a(2) = v'(2) \approx \frac{v(3) - v(1)}{3-1}$

$a(2) \approx -1 \frac{m}{\sec^2}$

As always, the difference quotient must clearly be stated. Since the problem asked for units, then the correct units must appear.

Once again, notation is very important. Please be sure to say a(2) NOT a(t) .

Also, there was no theorem involved in this problem, nor was a tangent line asked for, so I was not sure what some people were attempting to do with the inclusion of these things.

(c) Show that there must be a time interval such that the acceleration, $a(t)$, is equal to zero

By the Mean Value Theorem, there is a $t, 0 < t < 6,$ such that $a(t) = v'(t) = \frac{v(6) - v(0)}{6-0}$. Since $v(0) = v(6)$ OR since $\frac{v(6) - v(0)}{6-0} = 0$, then there must exist an $a(t) = 0$ on $0 < t < 6$.

OR

Since $v(0) = v(6)$ then by Rolle's Theorem there is a $t, 0 < t < 6,$ such that $v'(t) = a(t) = 0$

No theorem = no points!

You must explicitly state that $v'(t) = a(t)$ in either part (b) and/or part (c)