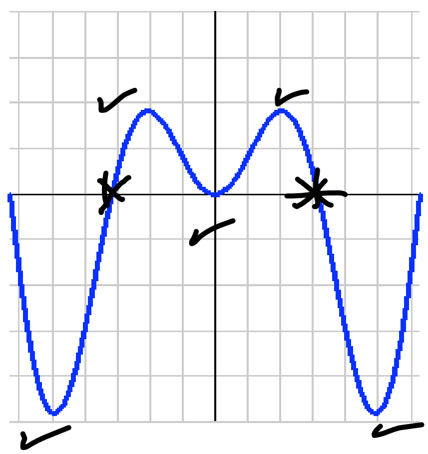
My Chapter 3 Quiz ____

Multiple Choice Directions

Solve each of the following problems, using the available space for your scratch work. After examining the form of the choices, decide which is the best of the choices given, and fill in the corresponding bubble on your Scantron form. No credit will be given for anything written in your scratch work. Do not spend too much time on any one problem.

1. The graph of $f'(x)$, the first derivative of f, is shown below. Determine which of the given statements about f must be true.



GRAPH of $f'(x)$

I. The graph of f has three relative extrema

II. The graph of f has two relative extrema

III. The graph of f has five points of inflection

(A) I only (B) II only (C) III only

(D) II and III only (E) I, II, and III

2. Let f be the function with derivative given by $f'(x) = x^2 - \frac{8}{x}$. On which of the following intervals is f decreasing?

(A) $(-\infty, 0)$ (B) $(0, 2)$ (C) $(2, \infty)$

(D) $(-\infty, 0)$ and $(2, \infty)$ (E) $(-\infty, 8)$

C.V $x = 0$ $f'(0)$ dne

$0 = x^2 - \frac{8}{x}$ CV $x = 2$

$(-\infty, 0)$ $(0, 2)$ $(2, \infty)$

$f' > 0$ $f' < 0$ $f' > 0$

3. If f is continuous on [2, 5] and differentiable on (2, 5) with $f(2) = -4$ and $f(5) = 14$, then of the following statements must be true?

I. $f(x) = 6$ has a solution in (2, 5) IVT?

II. $f'(x) = 6$ has a solution in (2, 5) MVT?

III. $f''(x) = 6$ has a solution in (2,5)

(A) I only (B) II only (C) I and II only

(D) I and III only (E) I, II, and III

By the MVT there is a, $2 < c < 5$,
such that $f'(c) = \frac{f(5) - f(2)}{5 - 2} = 6$

4. Let f be a twice-differentiable function whose second derivative is given by

$f''(x) = x^2(x+3)(x-7)$. Which of the following is true?

(A) f has three points of inflection

(B) f is always concave down

(C) f is always concave up

(D) f has two points of inflection

(E) f has a point of inflection at the point (0, 0)

POSSIBLE POI

$x = 0, x = -3, x = 7$

$(-\infty, -3)$ $(-3, 0)$ $(0, 7)$ $(7, \infty)$

$f'' > 0$ ✓ $f'' < 0$ $f'' < 0$ ✓ $f'' > 0$

5. A particle is moving along a straight line. The position function for time $t \geq 0$ is given by $s(t) = t^3 - 6t^2 + 12t - 8$. For what interval(s) is the speed of the particle increasing?

(A) $t > 2$

(B) $0 \leq t < 3$

(C) $t \geq 0$

(D) $0 \leq t < 1$ and $t > 2$

(E) Speed is never decreasing

$s'(t) = v(t) = 3t^2 - 12t + 12$

$0 = 3(t-2)^2$

$(0, 2)$ $(2, \infty)$

$v > 0$ $v > 0$

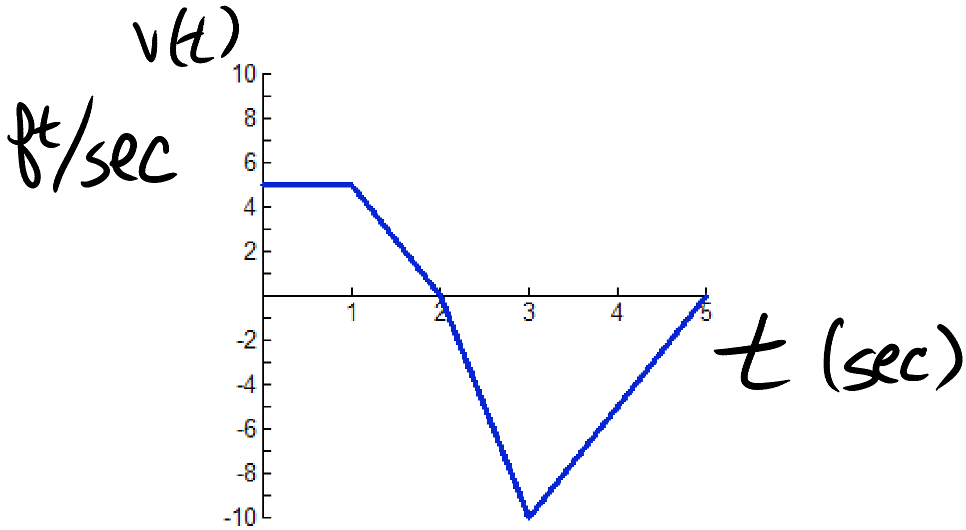
$s''(t) = a(t) = 6t - 12$

$0 = 6(t-2)$

$(0, 2)$ $(2, \infty)$

$a < 0$ $a > 0$

For questions 6, 7 and 8 use the graph below



The graph above shows the velocity of a particle moving along a horizontal line during the time interval $0 \le t \le 5$ seconds.

SPEED = $|v(t)|$

6. The particle attains it maximum speed when $t =$

(A) 0 (B) 1 (C) 2

(D) 3 (E) 5

7. At what time is the particle furthest to the right?

(A) 0 (B) 1 (C) 2

(D) 3 (E) 5

8. For what time interval is the acceleration of the particle positive?

(A) 0 < t < 1 (B) 1 < t < 2 (C) 0 < t < 2

(D) 2 < t < 3 (E) 3 < t < 5

v(t) increasing

9. The absolute minimum value of $f(x) = x^2 + \frac{2}{x}$ on the interval $\frac{1}{2} \leq x \leq 2$ is

(A) $\frac{1}{2}$

(B) 1

(C) 3

(D) 4

(E) 5

$f'(x) = 2x - \frac{2}{x^2}$

C.V. at $x=1$

$f(\frac{1}{2}) = 4.25$

$f(1) = 3$

$f(2) = 5$

10. Suppose $f'(x) = x(x-2)^2(x+3)$. Which of the following statement(s) is/are true?

I. f has a relative maximum at $x = -3$

II. f has a relative minimum at $x = 0$

III. f has neither a relative maximum nor a relative minimum at $x = 2$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

$(-\infty, -3)$ $(-3, 0)$ $(0, 2)$ $(2, \infty)$

$f' > 0$ $f' < 0$ $f' > 0$ $f' > 0$

rel max at $x = -3$

rel min at $x = 0$

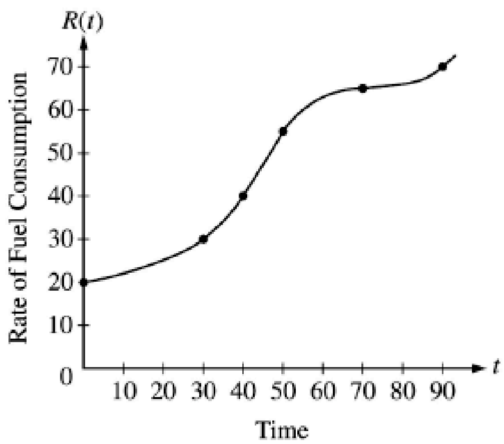
neither

End of Quiz. Please make sure that your name is on the quiz and the Scantron and that you have carefully filled out your Scantron.

If time we will finish these problems:

Rest of the Tan Line Problems

4.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$ for the time interval $0 \le t \le 90$ minutes are shown above.

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that led to your answer. Indicate units of measure.

$$R'(45) \approx \frac{R(50)-R(40)}{50-40}$$

$$= \frac{55-40}{10}$$

$$= 1.5 \ \frac{gal}{min^2}$$

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6. Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$. POINT

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f''(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

$$\frac{dy}{dx} = xy^3$$

FIRST DERIVATIVE

$$\frac{d^2y}{dx^2} = y^3(1+3x^2y^2)$$

SECOND DERIVATIVE

$$m_{TAN} = \left.\frac{dy}{dx}\right|_{(1,2)} = 8$$

$$y-2 = 8(x-1)$$

LINEAR APPROX.
OR
LOCAL LINEARIZATION

let $x = 1.1$

$$y - 2 = 8(1.1-1)$$

$$y - 2\ 8$$

$$f(1.1) \approx 2.8$$

FOR CONCAVITY

$$\left.\frac{d^2y}{dx^2}\right|_{(1,2)} > 0$$

for $1 < x < 1.1$
$f'' > 0$ Hence f IS CONCAVE up AND the Tangent LINE LIES BELOW the CURVE Hence APPROXIMATION $<$ ACTUAL value