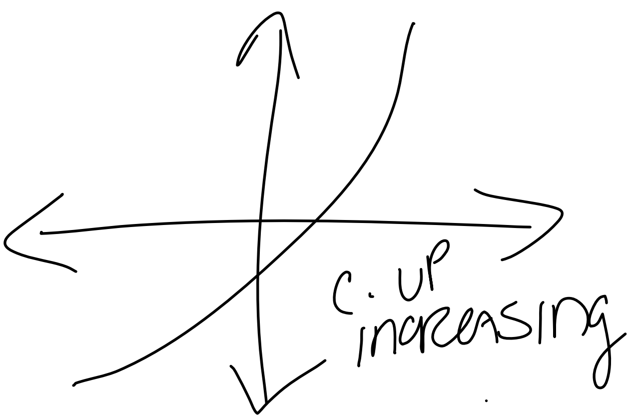
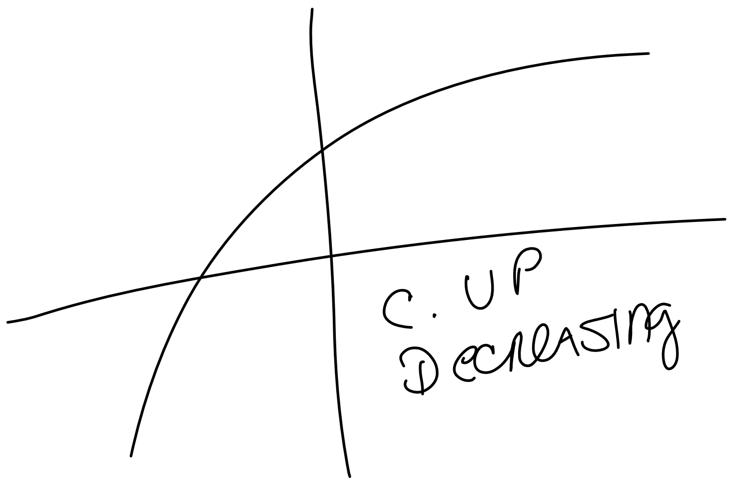
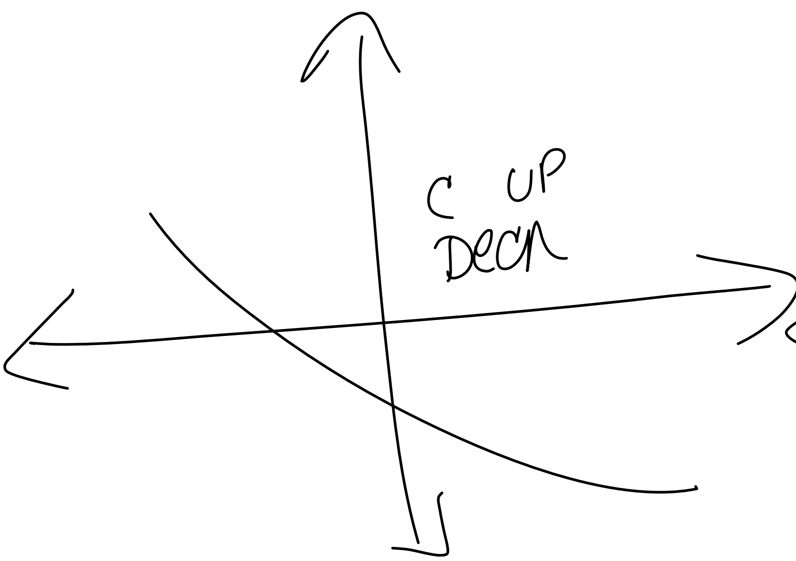
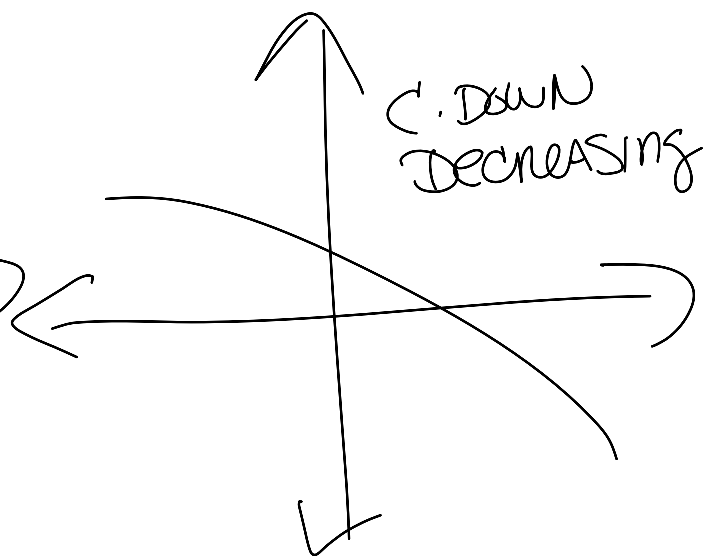
Let's see how much we learned from Chapter 3 so far:

1. Draw a function that is concave up increasing
2. Draw a function that is concave down increasing
3. Draw a function that is concave up decreasing
4. Draw a function that is concave down decreasing









5. Draw the graph of a function with the following attributes:

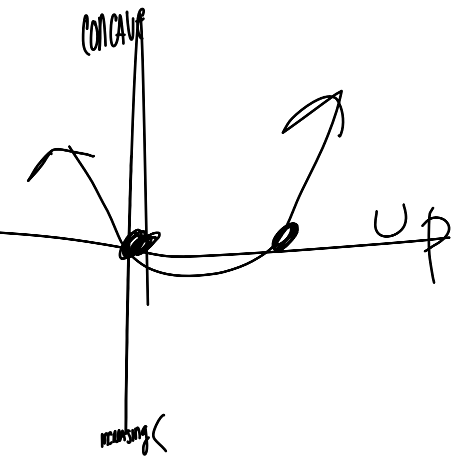
$f(0) = f(2) = 0$ Points

$f'(x) < 0$ if $x < 1$ f decreasing

$f'(1) = 0$

$f'(x) > 0$ if $x > 1$ f ↗ OR

$f''(x) > 0$ TAN



6. Let $f(x) = x^4 - 4x^3$

(a) Find $f'(x)$ and $f''(x)$

(b) Find all critical values

(c) Find all relative extrema

(d) Find all points of inflection

Justify completely.

$f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

let $f'(x) = 0$ to find c.v.

$0 = 4x^2(x-3)$ c.v. at $x=0$, $x=3$ because $f'(0) = f'(3) = 0$

FIRST DERIVATIVE TEST

$(-\infty, 0)$ $\quad$ $(0, 3)$ $\quad$ $(3, \infty)$

$f' < 0$ $\quad$ $f' < 0$ $\quad$ $f' > 0$

at $x = 3$ f' changes from negative to positive values. Hence f has a relative minimum at $x = 3$

Possible P of I $\quad$ let $f''(x) = 0$

$$0 = 12x(x-2)$$

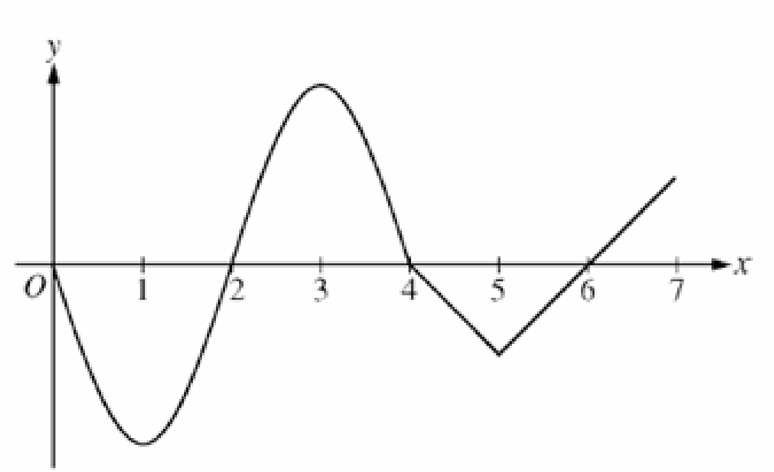
$(-\infty, 0)$ $\quad$ $(0, 2)$ $\quad$ $(2, \infty)$

$f'' > 0$ $\quad$ $f'' < 0$ $\quad$ $f'' > 0$

at $x = 0$ f'' changes from positive to negative values hence f has a point of inflection at $x = 0$

at $x = 2$ f'' changes from negative to positive values hence f has a point of inflection at $x = 2$

7. The graph below is the graph of $f'(x)$, the first derivative



Graph of f'

graph of $f'(x)$

Find the following:

The intervals where the graph of f is increasing or decreasing

The critical values of f

The relative extrema of f

The intervals where the graph of f is concave up or concave down and the points of inflection of f

$f' > 0$ on $(2,4)$, $(6,7]$ hence f is increasing on these intervals

$f' < 0$ on $(0,2)$, $(4,6)$ hence f is decreasing on these intervals

f has critical values at $x=2$, $x=4$, $x=6$ because $f'(x)=0$

at these values of x

d) maybe $x=0$ because it looks like $f'(0)=0$

at $x=2$ f' changes from negative to positive values hence f has a relative minimum at $x=2$

at $x=4$ f' changes from positive to negative values hence f has a relative maximum at $x=4$

at $x=6$ f' changes from negative to positive values hence f has a relative minimum at $x=6$

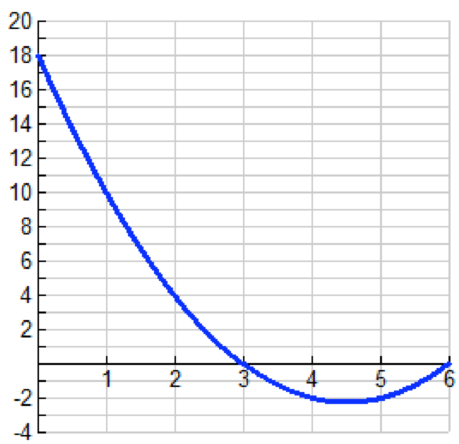
f' is decreasing on $(0,1)$, $(3,5)$ so f is concave down on these intervals

f' is increasing on $(1,3)$, $(5,7)$ so f is concave up on these interval

at $x=1$ f' changes from decreasing to increasing hence f has a point of inflection at $x=1$

at $X=3$ f' changes from increasing to decreasing hence f has a point of inflection at $X=3$
at $X=5$ f' changes from decreasing to increasing so f has a point of inflection at $X=0$

8. The graph below is the graph of $f(X)$



$f(3)=0$ $f'(3)<0$
$f''(3)>0$

$f'(3)< f(3) < f''(3)$

Compare the values of $f(3)$, $f'(3)$, $f''(3)$

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.

(a) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$?

(b)Use the data from the table to find an approximation of $R'(6)$.

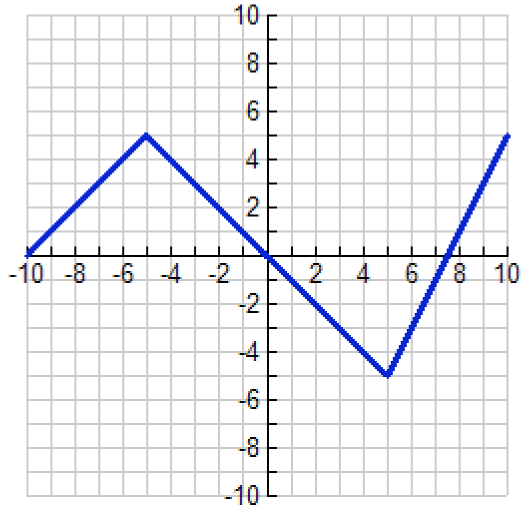
By the MVT there is a t, $0 < t < 24$, such that $R'(t) = \dfrac{R(24) - R(0)}{24 - 0}$

since $\dfrac{R(24) - R(0)}{24} = 0$ then there exists an $R'(t) = 0$ on $0 < t < 24$.

(b) $R'(6) \approx \dfrac{R(6) - R(3)}{6 - 3}$ OR $R'(6) \approx \dfrac{R(9) - R(6)}{9 - 6} = \dfrac{11.2 - 10.8}{3}$

3

10.



graph of $h'(x)$

The graph above is the graph of $h'(x)$, the first derivative of h.
Find all relative extrema and points of inflection for the graph of h. Justify completely.

at $X=0$ h' changes from positive to negative values hence h has a relative maximum at $X=0$

at $X=7.5$ h' changes from negative to positive values hence h has a relative minimum at $X=7.5$

at $X=-5$ h' changes from increasing to decreasing hence h has a point of inflection at $X=-5$

at $x=5$ f' changes from decreasing to increasing hence f has a point of inflection at $x=5$

11. A differentiable function f has the property that $f(7)=3$ and $f'(7)=-2$. What is the estimate for $f(7.1)$ using the local linearization approximation for f at $x=7$

$$y-3=-2(x-7)$$

let $x=7.1$

$$y-3=-2(7.1-7)$$

$$y-3=-.2$$

$$y=2.8$$

$$f(7.1)\approx 2.8$$