

Let's sing first!

Increasing and Decreasing Functions

A function f is increasing on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies

$$f(x_1) < f(x_2)$$

Likewise, for decreasing

How to prove a function is increasing or decreasing
using Calculus [not our eyes or calculator]

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b)

- (1) If $f'(x) > 0$ for all x in (a, b) , then f is increasing on $[a, b]$
- (2) If $f'(x) < 0$ for all x in (a, b) , then f is decreasing on $[a, b]$
- (3) If $f'(x) = 0$ for all x in (a, b) , then f is constant

~ horizontal line

A function is *strictly monotonic* if it is either increasing on an entire interval, or decreasing on an entire interval

Let's look at some functions [without our TI]

Let $f(x) = 4x - x^2$ on $[-4, 4]$

Step 1: find $f'(x)$ and the critical values for this interval

$$f'(x) = 4 - 2x$$

$$\text{Let } f'(x) = 0$$

$$0 = 4 - 2x$$

Hence, our only critical value is $x = 2$

$$f'(x) = 4 - 2x$$

Step 2: test intervals

$$-4 < x < 2$$

$$\text{Let } x = 0$$

$$f'(0) = 4$$

$f'(x) > 0$ on this interval

f is increasing on this int.

$$2 < x < 4$$

$$\text{Let } x = 3$$

$$f'(3) = -2$$

$f'(x) < 0$ on this interval

f is decreasing on this int.

See Guidelines for Finding Intervals on Which a Function is Increasing or Decreasing on page 180

♪ If you learned a “number line” approach in a previous class, then do NOT use it. The AP requires a thoughtful analysis which includes nouns and verbs!

And now, ***The First Derivative Test***

Let c be a critical number [value] of a function f that is continuous on an open interval I containing c . If f is differentiable on the interval, except possibly at c , then $f(c)$ can be classified as follows:

If $f'(x)$ changes from negative to positive values at $x=c$, then f has a relative minimum at $(c, f(c))$

Consider $y = x^2 + 4$ on the interval $(-5, 5)$

My analysis:

The function is both continuous and differentiable on the interval [yay!]

Now let's find the critical number(s):

$$0 = 2x$$

$$y' = 2x$$

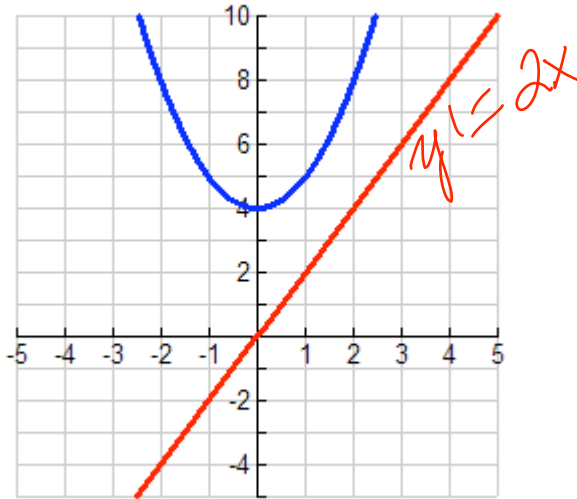
Our only critical number is $x=0$

Interval	$-5 < x < 0$	$0 < x < 5$
Test Value	$x = -1$	$x = 1$
Sign of $f'(x)$	$f'(-1) = -2$ $-2 < 0 \quad f' < 0$	$f'(1) = 2$ $2 > 0 \quad f' > 0$
Conclusion	f is <i>decr</i>	f is <i>incr</i>

AND $f'(x)$ is changing from negative to positive values at $x=0$. Hence, $(0, 4)$ is a relative minimum for $y = x^2 + 4$

Here is a graph of $y = x^2 + 4$ and $y' = 2x$

$$y = x^2 + 4$$



you may NOT use a graph for your justification

If $f'(x)$ changes from positive to negative values at $x = c$, then f has a relative maximum at $(c, f(c))$

Consider $f(x) = 5 - 2x - x^2$ on the interval $(-4, 4)$

My analysis:

The function is both continuous and differentiable on the interval.

Now let's find any critical numbers.

$$f'(x) = -2 - 2x$$

$$0 = -2 - 2x$$

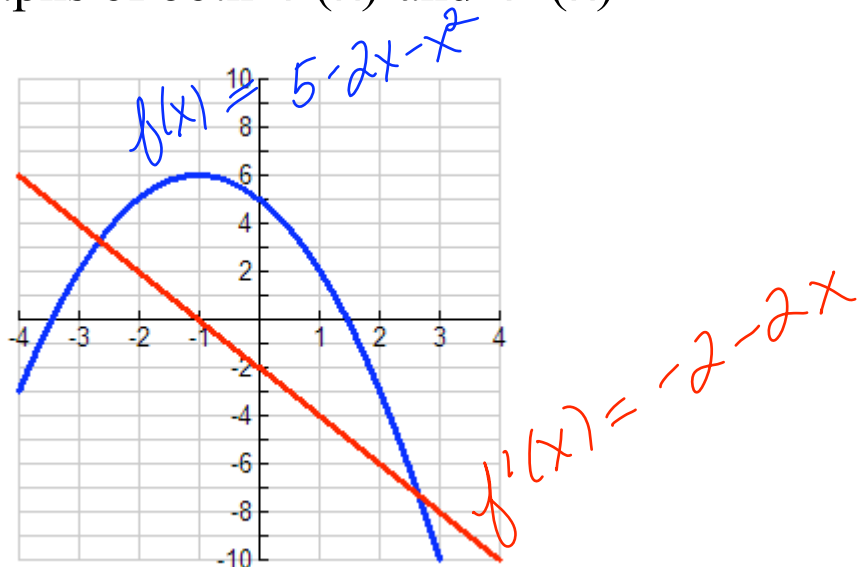
Our only critical number is $x = -1$

Interval	$-4 < x < -1$	$-1 < x < 4$
Test Value	$x = -2$	$x = 0$
Sign of $f'(x)$	$f'(-2) = 2$ $2 > 0$ $f' > 0$	$f'(0) = -2$ $-2 < 0$ $f' < 0$

Conclusion	f is	f is
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Since $f'(x)$ changes from positive to negative values at $x = -1$, then the graph of $f(x)$ has a relative maximum at the point $(-1, 6)$. The relative maximum value is 6.

Here are the graphs of both $f(x)$ and $f'(x)$



If $f'(x)$ is positive on both sides of $x = c$, then $f(c)$ is neither a relative minimum or a relative maximum.

Consider $f(x) = x^3 + 3$ on $(-3, 3)$

The function is both continuous and differentiable on the interval.

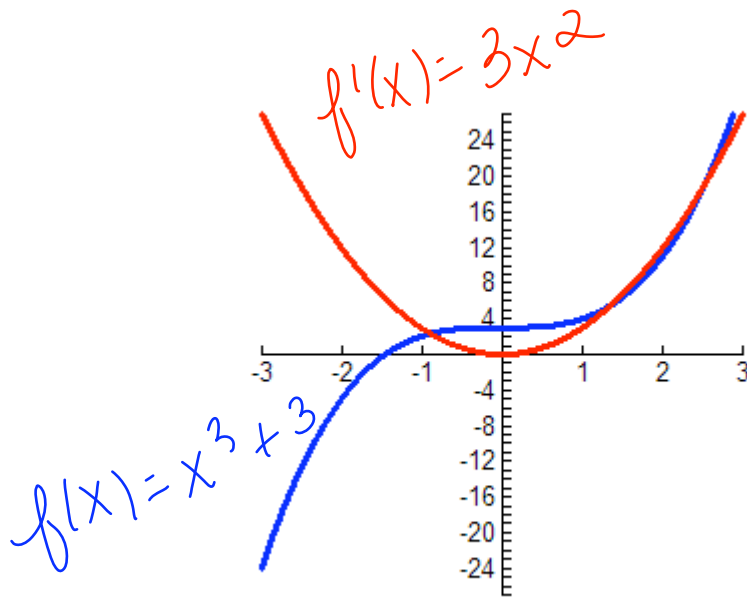
$$f'(x) = 3x^2$$

Our only critical value is at $x=0$

Interval	$-3 < x < 0$	$0 < x < 3$
Test Value	$x = -1$	$x = 1$
Value of $f'(x)$	$f'(-1) = 3$ $3 > 0$ $f' > 0$	$f'(1) = 3$ $3 > 0$ $f' > 0$
Conclusion	f is increasing	f is increasing

In this case, there is neither a relative minimum or a relative maximum at our critical value.

Here are the graphs of $f(x)$ and $f'(x)$



Let's try to analyze some functions WITHOUT our calculators.

$$(-\infty, \infty)$$

Let $f(x) = x^2 + 8x + 10$

Our mission:

- (a) find all critical numbers
- (b) find all open intervals where the function is increasing or decreasing
- (c) apply the First Derivative test to identify all relative extrema

$f'(x) = 2x + 8$

$0 = 2x + 8$

Let $f'(x) = 0$

Our only critical number is $x = -4$

My analysis

Interval	$(-\infty, -4)$	$(-4, \infty)$
Test Value	$x = -5$	$x = 0$
Sign of $f'(x)$	$f' < 0$	$f' > 0$
Conclusion	f decreasing	f increasing

Statement about any extrema:

At $x = -4$ f' is changing from negative to positive values Hence f has a rel min at $x = -4$
 Our rel min value is -6

Let $f(x) = (x-2)^2(x-1)$

$$f'(x) = 2u \frac{du}{dx} (x-1) + u^2 (1)$$

$$= 2(x-2)(x-1) + (x-2)^2$$

$$= (x-2)[2x-2 + x-2]$$

$$u = x-2$$

$$\frac{du}{dx} = 1$$

Critical value(s): $= (x-2)(3x-4)$

$$x = 2, x = \frac{4}{3}$$

My analysis

Interval	$(-\infty, \frac{4}{3})$	$(\frac{4}{3}, 2)$	$(2, \infty)$
Test Value	$x = 0$	$x = 1.5$	$x = 5000000$
Sign of $f'(x)$	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion	f incr	f decr	f incr

\downarrow rel max \downarrow rel min

Statement about extrema:

At $x = \frac{4}{3}$ f' changes from positive to negative values hence f has a rel max at $x = \frac{4}{3}$

At $x = 2$ f' changes from negative to positive values hence f has a rel min at $x = 2$

Homework: page 186 #19, 21, 23, 27, 35

Do as we did in class
need to see work
see statements