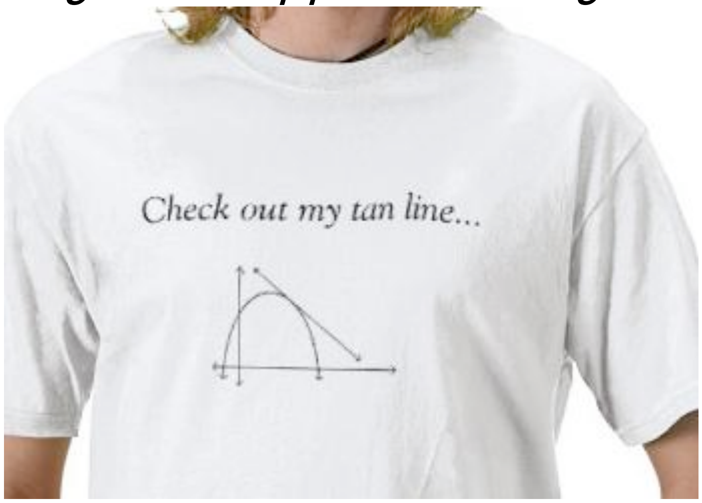
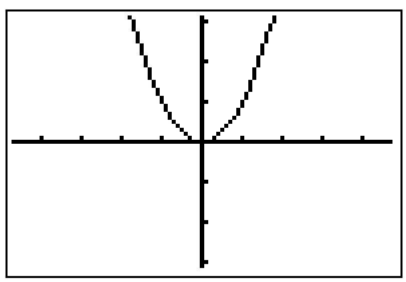
Tangents, smangents!
[Estimating a tangent or approximating with a tangent]



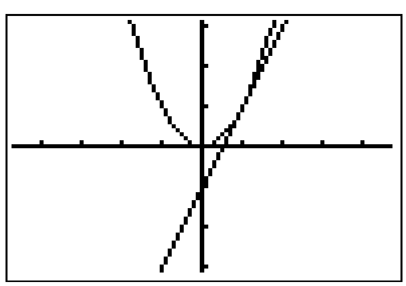
Let's start by graphing my favorite function
$y = x^2$ and use the "Zoom Decimal" window



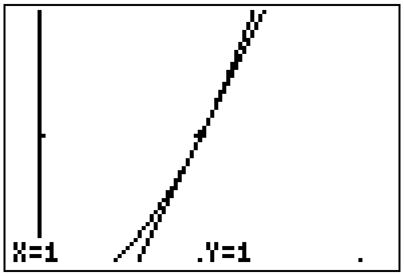
$f'(x) = 2x$
$f'(1) = 2$

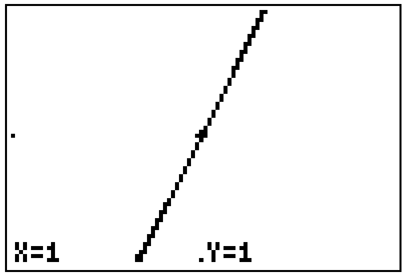
Now find and then graph the tangent to the graph at the point (1, 1)

$y - 1 = 2(x - 1)$
$y = 2x - 1$



Now let's zoom in at the point (1, 1)





What do you notice?

Differentiable functions are "locally linear". If a function is differentiable at the point $(a, f(a))$, then it resembles its own tangent line at the point of tangency.
[Remember that at the point (0, 0) on the graph of $y = |x|$, the "sharp point" remains sharp no matter how many times we "zoomed in".]

Why is this useful?
It is easier to evaluate values of a linear function. So for values of x near the point of tangency, we can just use the tangent line equation to approximate values of the function. This is even more useful when you don't have a function to work with.

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$. f is concave up

Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Will this approximation be an under- or over-estimate of the actual value?

Point: $(1, -4)$

m_{Tan}: $(1, 5)$

$y + 4 = 5(x - 1)$

Now let $x = 1.2$

$y + 4 = 5(1.2 - 1)$

$y = -3$

$f(1.2) \approx -3$

For $-1.5 \le x \le 1.5$ f is concave up because $f''(x) > 0$. Hence the tangent line below the curve so this would be an under-estimate for $f(1.2)$

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Let f be a differentiable function. Estimate $f(2.1)$ given that $\underbrace{f(2)=1}_{\text{Point}}$ and $\underbrace{f'(2)=3}_{m_{tan}}$.

$y-1=3(x-2)$

let $x=2.1$

$y-1=3(2.1-2)$

$y=1.3$

$f(2.1)\approx 1.3$

Find the equation of the tangent line to $y=\sin x$ at the origin and use it to find an estimation of $\sin(0.12)$
[After finding the estimated value, check to see how close it is to the actual value]

Point: $(0,0)$

m_{tan} $y'=\cos x$

$y'(0)=1$

$y-0=1(x-0)$

let $x=.12$

$y=.12$

OUR TI SAYS $\sin(0.12)\approx .1197$

Using tables to find estimations

Let $y(t)$ represent the temperature of a pie that has been removed from a 450°F oven and left to cool in a room with a temperature of 72°F, where y is a differentiable function of t. The table below shows the temperature recorded every five minutes.

t (min)	0	5	10	15	20	25	30
$y(t)$ (°F)	450	388	338	292	257	226	200

Use data from the table to find an approximation for $y'(18)$ and explain the meaning of $y'(18)$ in terms of the temperature of the pie. Show the computations that lead to your answer and indicate units of measure.

$$y'(18) \approx \frac{y(20) - y(15)}{20 - 15} \frac{°F}{min}$$

$$= \frac{-35}{5} \frac{°F}{min}$$

$$= -7 \frac{°F}{min}$$

at $t=18$ The PIE IS COOLING AT A RATE OF APPROXIMATELY 7°F/min

Let $y(t)$ represent the population of a town over a 20-year period, where y is a differentiable function of t. The table below shows the population recorded at selected times.

t (yrs)	0	4	10	13	20
$y(t)$ (people)	2500	2724	3108	3697	4283

Use data from the table to find an approximation for $y'(12)$, and explain the meaning of $y'(12)$ in terms of the population of the town. Show the computations that lead to your answer.

$$y'(12) \approx \frac{y(13)-y(10)}{13-10} \quad \frac{\text{PEEPS}}{\text{YR}}$$

$$\approx \frac{589}{3} \ \frac{\text{Peeps}}{\text{yr}}$$

at $t=12$ the POPULATION grew at a rate of $\approx 589/3$ P/yr

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2. The function g is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$.

(a) Find all values of x in the interval $0.12 \le x \le 1$ at which the graph of g has a horizontal tangent line.

(b) On what subintervals of $(0.12, 1)$, if any, is the graph of g concave down? Justify your answer.

(c) Write an equation for the line tangent to the graph of g at $x = 0.3$.

(d) Does the line tangent to the graph of g at $x = 0.3$ lie above or below the graph of g for $0.3 < x < 1$? Why?

(a) $g'(x) = 0$ use our TI to graph $g'(x)$

hor tan at $x \approx 0.163$

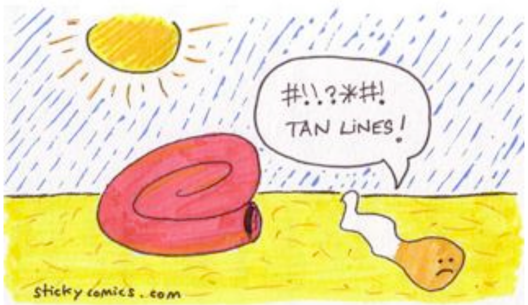
$x \approx 0.359$

(b) g is CONCAVE DOWN [LIKE A FROWN]
on $(0.129, 0.223)$
Because $g''(x) < 0$ on this interval

(c) $y = 1.546 - 0.472(x - .3)$

(d) at $x = 3$ for $.3 < x < 1$
$g''(x) > 0$ ~~so~~ g is c-up
and the tan line lies below the curve

Pesky "Tan" Lines *20102011tanlines.doc*



From: stickycomics.com

Available at www.snorgtees.com

Examples from class:

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
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****

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   (c) Write an equation for the line tangent to the graph of g at $x = 0.3$.

   (d) Does the line tangent to the graph of g at $x = 0.3$ lie above or below the graph of g for $0.3 < x < 1$? Why?

Homework Problems
Please do these problems on a separate piece of paper
Show ALL steps

1. *Consider the differential equation* $\frac{dy}{dx} = \frac{x}{y}$ *where* $y \neq 0$. *Write the equation of the tangent line to the solution curve that passes through the point* $(3, -1)$ *and use it to approximate* $f(2.9)$

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2. *Let* f *be the function given by* $f(x) = x^3 - 6x^2 + 7x + 3$. *Write the equation of the tangent line to the graph at* $x = 4$ *and use it to approximate* $f(4.2)$. *Then, determine whether this approximation is an under- or over-estimate by using the second derivative.*

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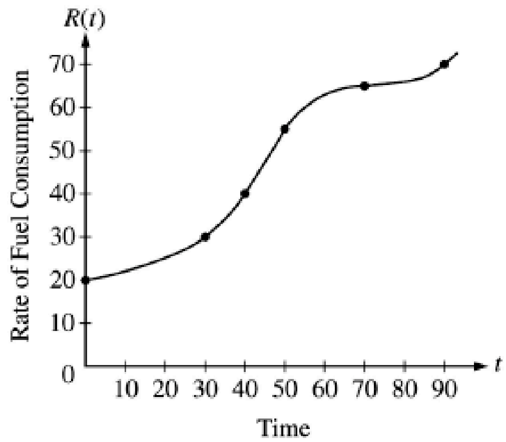
3.

Car *A* has positive velocity $v_A(t)$ as it travels on a straight road, where v_A is a differentiable function of *t*. The velocity is recorded for selected values over the time interval $0 \leq t \leq 10$ seconds, as shown in the table below.

t (sec)	0	2	5	7	10
$v_A(t)$ (ft/sec)	0	9	36	61	115

a. Use data from the table to approximate the acceleration of Car *A* at $t = 8$ seconds. Indicate units of measure.

4.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$ for the time interval $0 \le t \le 90$ minutes are shown above.

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that led to your answer. Indicate units of measure.

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6. Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.