1. The function $g$ is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and

$g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$ [This is a calculator question]

(a) Find all values of $x$ in the interval $0.12 \le x \le 1$ at which the graph has a horizontal tangent line.
(b) On what subintervals of $(0.12, 1)$, if any, is the graph of $g$ concave down. Justify.
(c) Write an equation for the line tangent to the graph of $g$ at $x = 0.3$
(d) Does the line tangent to the graph of $g$ lie above or below the graph of $g$ for $0.3 < x < 1$? Justify.

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2. Let $f$ be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers $x$, where $f(3) = 25$.

Find $f''(x)$

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3. A particle moves along the $x$- axis so that its velocity at time $t$ is given by

$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$ [Calculator]

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Show all steps that lead to your conclusion.

(b) Find all times $t$ in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer [with Calculus]

# This is a calculator problem. So I used my trusty TI.

$a(t) = v'(t)$ and $a(2) \approx 1.5875$. $v(2) \approx -2.72789$ At t = 2, v(2) < 0 and a (2) > 0. Hence, the speed is decreasing at t = 2.

At $t \approx 2.5066$ v(t) changes from negative to positive values. Hence, the particle changes direction at $t \approx 2.5066$.

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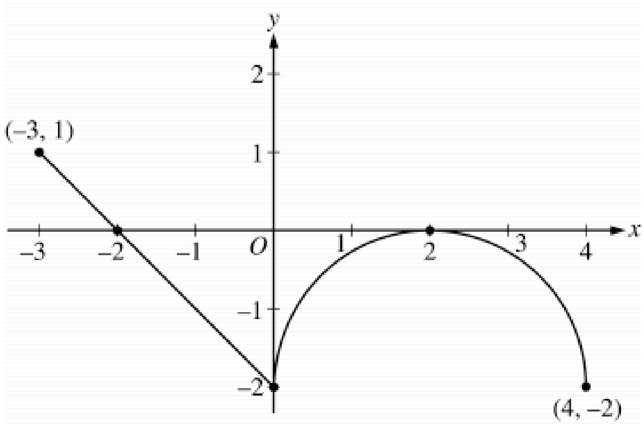

Graph of $f'$

4. Let $f$ be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of $f'$, the derivative of $f$, consists of one line segment and a semicircle, as shown above.

(a) On what intervals, if any, is $f$ increasing? Justify your answer.

(b) Find the $x$-coordinate of each point of inflection of the graph of $f$ on the open interval $-3 < x < 4$. Justify your answer.

(a) $f'(x) > 0$ for $[-3, 2)$. Hence, f is increasing on $[-3, -2]$

Please notice the notation.

(b) $f'$ changes from decreasing to increasing at x = 0. $f'$ changes from increasing to decreasing at x = 2. Hence, $f$ has points of inflection at x = 0 and x=2.

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5.

For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time $t$ days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

$R(6) \approx 4.438$ Since $R(6) > 0$ then the number of mosquitoes is increasing at t = 6.

$R'(6) \approx -1.913$ Since $R'(6) < 0$ then the number of mosquitoes is increasing at a decreasing rate at t = 6.

6. A particle moves along a horizontal line with a positive velocity $v(t)$, where $v$ is a differentiable function of $t$. The time $t$ is measured in seconds, and the velocity is measured in cm/sec. The velocity of the particle at selected times is given in the table below.

| $t$ (sec) | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| $v(t)$(cm/sec) | 37 | 17 | 5 | 1 | 6 | 17 | 38 |

a. Based on the values in the table, what is the smallest number of times at which the velocity of the particle could equal 20 cm/sec in the open interval $0 < t < 12$ seconds? Justify your answer.

b. Based on the values in the table, what is the smallest number of times at which the acceleration of the particle could equal zero in the open interval $0 < t < 12$ seconds? Justify your answer.

c. Find the average acceleration of the particle over the time interval $8 \le t \le 10$ seconds. Show the computations that lead to your answer, and indicate units of measure.

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6a

Two times. By the Intermediate Value Theorem there is a $t_1, 0 < t_1 < 2$ such that $v(0) > v(t_1) > 0$ or $17 < v(t_1) < 37$. Also, by the IVT, there is a $t_2, 10 < t_2 < 12$, such that $v(10) < v(t_2) < v(12)$ or $17 < v(t_2) < 38$. Hence, there is at least two times that v(t) = 20 cm/sec

6b

By the Mean Value Theorem there is a $t_3, 2 < t_3 < 10$, such that $a(t_3) = v'(t_3) = \frac{v(10) - v(2)}{10-2}$. Since $\frac{v(10) - v(2)}{8} = 0$, then there must exist an $a(t_3) = 0$ for $2 < t_3 < 10$.

6c

Average acceleration on [8, 10 ] = $\frac{v(10) - v(8)}{10-8}$ which equals $\frac{11}{2} \frac{cm}{\sec^2}$

7.

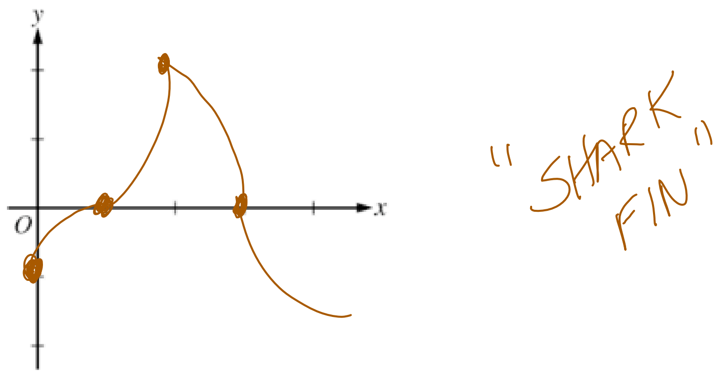
| $x$ | 0 | $0 < x < 1$ | 1 | $1 < x < 2$ | 2 | $2 < x < 3$ | 3 | $3 < x < 4$ |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | −1 | Negative | 0 | Positive | 2 | Positive | 0 | Negative |
| $f'(x)$ | 4 | Positive | 0 | Positive | DNE | Negative | −3 | Negative |
| $f''(x)$ | −2 | Negative | 0 | Positive | DNE | Negative | 0 | Positive |

4. Let $f$ be a function that is continuous on the interval $[0, 4)$. The function $f$ is twice differentiable except at $x = 2$. The function $f$ and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of $f$ do not exist at $x = 2$.

   (a) For $0 < x < 4$, find all values of $x$ at which $f$ has a relative extremum. Determine whether $f$ has a relative maximum or a relative minimum at each of these values. Justify your answer.

   (b) On the axes provided, sketch the graph of a function that has all the characteristics of $f$.

   **(Note: Use the axes provided in the pink test booklet.)**

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7a.

At $x = 2$ $f'$ changes from positive to negative values. Hence $f$ has a relative maximum at x = 2.