

1. The function g is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and

$$g''(x) = \left(1 - \frac{1}{x^2}\right) \cos\left(x + \frac{1}{x}\right) \quad \text{[This is a calculator question]}$$

- (a) Find all values of x in the interval $0.12 \leq x \leq 1$ at which the graph has a horizontal tangent line.
- (b) On what subintervals of $(0.12, 1)$, if any, is the graph of g concave down. Justify.
- (c) Does the line tangent to the graph of g lie above or below the graph of g for $0.3 < x < 1$? Justify.

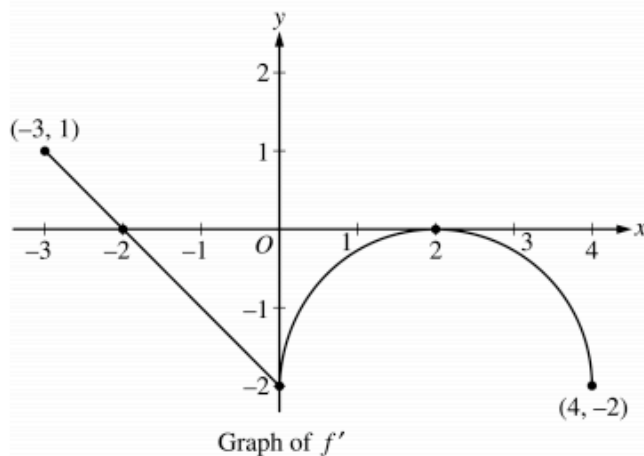
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2. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x , where $f(3) = 25$.

Find $f''(x)$

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3. A particle moves along the x -axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right) \quad \text{[Calculator]}$$

- (a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Show all steps that lead to your conclusion.
- (b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer [with Calculus]



4. Let f be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of f' , the derivative of f , consists of one line segment and a semicircle, as shown above.
- On what intervals, if any, is f increasing? Justify your answer.
 - Find the x -coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

5.

For $0 \leq t \leq 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t} \cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

- Show that the number of mosquitoes is increasing at time $t = 6$.
- At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

6. A particle moves along a horizontal line with a positive velocity $v(t)$, where v is a differentiable function of t . The time t is measured in seconds, and the velocity is measured in cm/sec. The velocity of the particle at selected times is given in the table below.

t (sec)	0	2	4	6	8	10	12
$v(t)$ (cm/sec)	37	17	5	1	6	17	38

- a. Based on the values in the table, what is the smallest number of times at which the velocity of the particle could equal 20 cm/sec in the open interval $0 < t < 12$ seconds? Justify your answer.
- b. Based on the values in the table, what is the smallest number of times at which the acceleration of the particle could equal zero in the open interval $0 < t < 12$ seconds? Justify your answer.
- c. Find the average acceleration of the particle over the time interval $8 \leq t \leq 10$ seconds. Show the computations that lead to your answer, and indicate units of measure.

7.

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

4. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f .

(Note: Use the axes provided in the pink test booklet.)

