Chapter Two AP Problems [the real deal]

Non-calculator free response

4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

$$\frac{d}{dx}x^2 + \frac{d}{dx}4y^2 = \frac{d}{dx}7 + \frac{d}{dx}\underline{3xy}$$

$$2x + 8y\frac{dy}{dx} = 3y + 3x\frac{dy}{dx}$$

$$\frac{dy}{dx}(8y-3x) = 3y-2x$$

$$\frac{dy}{dx} = \frac{3y-2x}{8y-3x}$$

let $\frac{dy}{dx} = 0$

$3y - 2x = 0$

let $x = 3$

$3y - 6 = 0$

$y = 2$

OR let $x = 3$

$x^2 + 4y^2 = 7 + 3xy$

$9 + 4y^2 = 7 + 9y$

$4y^2 - 9y + 2 = 0$

$(4y-1)(y-2) = 0$

need to see if $\frac{dy}{dx} = 0$

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CHECK TO SEE IF (3,2) IS ON THE CURVE

$x^2 + 4y^2 = 7 + 3xy$

$9 + 16 = 7 + 18$

P: (3,2)

6. Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\frac{dy}{dx} = \frac{-(x+1)}{2(y^3+1)}$.

$y - 1 = \frac{1}{4}(x+2)$ $\qquad \left.\frac{dy}{dx}\right| = \frac{1}{4}$

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$. $\quad (-2,1)$

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

$$x^2 + 2x + y^4 + 4y = 5$$

$$\frac{d}{dx}x^2 + \frac{d}{dx}2x + \frac{d}{dx}y^4 + \frac{d}{dx}4y = \frac{d}{dx}5$$

$$2x + 2 + 4y^3\frac{dy}{dx} + 4\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(4y^3+4) = -2x-2$$

$$\frac{dy}{dx} = \frac{-2x-2}{4y^3+4}$$

$$\frac{dy}{dx} = \frac{-2(x+1)}{4(y^3+1)}$$

$$= \frac{-(x+1)}{2(y^3+1)}$$

$\frac{dy}{dx}$ is undefined

IF $2(y^3+1)=0$ and $-(x+1) \neq 0$ ✲✲✲✲

let $y = -1$

$x^2 + 2x + 1 - 4 = 5$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

Points:

$(-4, -1)$

$(2, -1)$

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Multiple-choice Non-calculator

If $x^2 + xy = 10$, then when $x = 2, \frac{dy}{dx} =$

(A) -3.5 (B) -2 (C) 2/7

(D) 1.5 (E) 3.5

Point: $4 + 2y = 10$ $y = 3$ $(2, 3)$

$\frac{d}{dx}x^2 + \frac{d}{dx}xy = \frac{d}{dx}10$

$2x + y + x\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x - y}{x}$

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If f(x)=tan(2x), then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4

(D) $4\sqrt{3}$ (E) 8

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$f'(x) = 2\sec^2(2x)$

$f'\left(\frac{\pi}{6}\right) = 2\left[\sec\frac{\pi}{3}\right]^2$

Multiple-Choice Calculator

The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $\frac{-(0.1)C}{\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$

find $\frac{da}{dt}$ $C = 2\pi r$

$a = \pi r^2$ $\frac{dr}{dt} = -0.1$

$\frac{d}{dt} a = \frac{d}{dt} \pi r^2$

$\frac{da}{dt} = 2\pi r \frac{dr}{dt}$

$= -0.1C$