Find the value of the constants a and b that make the function differentiable at $x=2$.

$$f(x)=\begin{cases} ax^4+5x, & x\le 2 \\ bx^2-3x, & x>2 \end{cases}$$

To be continuous we need $f(2)=\lim_{x\to 2} f(x)$

$$f(2)=16a+10$$

$$\lim_{x\to 2^-} f(x)=16a+10 \qquad \lim_{x\to 2^+} f(x)=4b-6$$

In order to have $\lim_{x\to 2} f(x)$ we will need $16a+10=4b-6$ OR $16a-4b=-16$

[Note: At this point we do not have enough information to determine the values of a or b

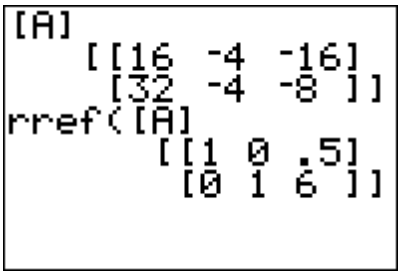
To be differentiable we will need $\lim_{x\to 2^-} f'(x)=\lim_{x\to 2^+} f'(x)$ where

$$f'(x)=\begin{cases} 4ax^3+5, & x\le 2 \\ 2bx-3, & x>2 \end{cases}$$

$$\lim_{x\to 2^-} f'(x)=32a+5 \qquad \lim_{x\to 2^+} f'(x)=4b-3$$

Hence, we will need $32a+5=4b-3$ OR $32a-4b=-8$

[At this point we have enough information to determine the values of a and b, namely, we have two equations with two unknowns.]



Using my trusty TI, Billy Bob, I have found that $a=0.5$ and $b=6$