Now that we know how to use the Chain Rule, we can do some AP-style derivatives.

Using Tables to Find Derivative Values [or "Oh no! My calculator can't do this!"]

1. Suppose that the function $f(x)$ and its first derivative have the following values at $x=0$ and $x=4$.

x	$f(x)$	$f'(x)$
0	25	-3
4	-10	7

Find the first derivative of the following combinations at the given value of x

(a) $\sqrt{x}\, f(x)$ at $x=4$ [Product Rule]

$$\frac{d}{dx}\sqrt{x}\, f(x)$$

$$= \frac{1}{2\sqrt{x}} f(x) + f'(x)\sqrt{x}$$

at $x=4$ $\quad \frac{1}{2\sqrt{4}}(-10) + 7\sqrt{4}$

(b) $\sqrt{f(x)}$ at $x=4$ [Chain Rule]

$u = f(x)$

$\frac{du}{dx} = f'(x)$

$$\frac{d}{dx}\sqrt{u}$$

$$= \frac{1}{2\sqrt{u}} \frac{du}{dx}$$

$$= \frac{1}{2\sqrt{f(x)}} f'(x)$$

at $x=4$ $\quad \frac{1}{2\sqrt{f(4)}} f'(4)$

ACK! my BAD

(c) $\dfrac{f(x)}{7+\cos x}$ at $x=4$ [Quotient Rule]

$$\frac{d}{dx}\frac{f(x)}{7+\cos x}$$

$$= \frac{(7+\cos x)\,f'(x) - f(x)[-\sin x]}{(7+\cos x)^2}$$

$$= \frac{(7+\cos 4)\,f'(4) - f(4)[-\sin 4]}{(7+\cos 4)^2}$$

(d) $7x + f^3(x)$ at $x=0$ [Sum and Chain Rule]

$$\frac{d}{dx}[7x+u^3]$$

$$u = f(x) \qquad \frac{du}{dx} = f'(x)$$

$$= \left[7 + 3u^2\frac{du}{dx}\right]$$

$$= [7 + 3f^2(x)f'(x)]$$

at $x = 0$

$$7 + 3f^2(0) \quad f'(0)$$

$$= 7 + 3(25^2)(-3)$$

(e) $\dfrac{1}{f(x)}$ at $x=0$ [Quotient Rule or Chain Rule]

CHAIN RULE

$$\frac{d}{dx}\frac{1}{u} \qquad u = f(x) \quad \frac{du}{dx} = f'(x)$$

$$= -\frac{1}{u^2}\frac{du}{dx}$$

$$= -\frac{f'(x)}{f^2(x)}$$

at $x=0$ $= \dfrac{3}{25^2}$

Quotient Rule

$$\frac{d}{dx}\frac{1}{f(x)}$$

$$= \frac{[f(x)](0) - f'(x)}{f^2(x)}$$

$$= \frac{3}{25^2}$$

An AP-style Multiple Choice Example

Let f be a differentiable function such that $f(7)=11$ and $f'(7)=-5$ and $g(x)=x^2 f(x)$. What would be the value of $g'(7)$? [Calculator question]

(A) 91
(B) 275
(C) -91
(D) 154
(E) -245

$$g(x) = x^2 f(x)$$

$$g'(x) = 2x\, f(x) + x^2 f'(x)$$

$$g'(7) = 2(7) f(7) + (7^2) f'(7)$$

Now let's decide which fancy-pants rule(s) we would need for some of the questions on the handout.

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Given the following information about differentiable functions $f(x)$ and $g(x)$ at $x=2$ determine the following:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	8	2	2π	e

(a) $\frac{d}{dx}[f(g(x))]$ at $x=2$ CHAIN

(b) $\frac{d}{dx}\left(\frac{1}{f(x)}\right)$ at $x=2$ QUOTIENT OR CHAIN

(c) $\frac{d}{dx}\left[\frac{g(x)}{f(x)}\right]$ at $x=2$

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$$\frac{d}{dx}\frac{g(x)}{f(x)} = \frac{f(x)g'(x) - g(x)f'(x)}{f^2(x)}$$

$$= \frac{(8)(e) - (2)(2\pi)}{8^2}$$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	8	2	5	$\frac{1}{3}$
3	3	-4	$-\frac{1}{3}$	$-\frac{8}{3}$

On a separate sheet of paper, find the following:

(a) $\frac{d}{dx}[2f(x)]$ at $x=2$ CMR

(b) $\frac{d}{dx}[f(x)+g(x)]$ at $x=3$ SUM

(c) $\frac{d}{dx}[f(x)\bullet g(x)]$ at $x=3$ PRODUCT

(d) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$ at $x=2$ QUOTIENT

(e) $\frac{d}{dx}[f(g(x))]$ at $x=2$ CHAIN

(f) $\frac{d}{dx}\left[\sqrt{f(x)}\right]$ at $x=2$ CHAIN $\sqrt{u}$

(g) $\frac{d}{dx}\left[\frac{1}{g(x)}\right]$ at $x=3$ QUOTIENT or CHAIN

Now let's do the "Harry Potter and the Deathly Derivatives" puzzle