Solutions for the Free Response Questions on the Chapter Test

FR#1

On the axes provided below, sketch a graph of a function that contains ALL of the following properties:

$$\lim_{x \to -5} f(x) = \infty$$

Vertical asymptote from both sides of x = - 5

$\lim_{x \to -1^-} f(x) = 5$ $\qquad \lim_{x \to -1^+} f(x) = -2$ $\qquad f(-1) = 3$

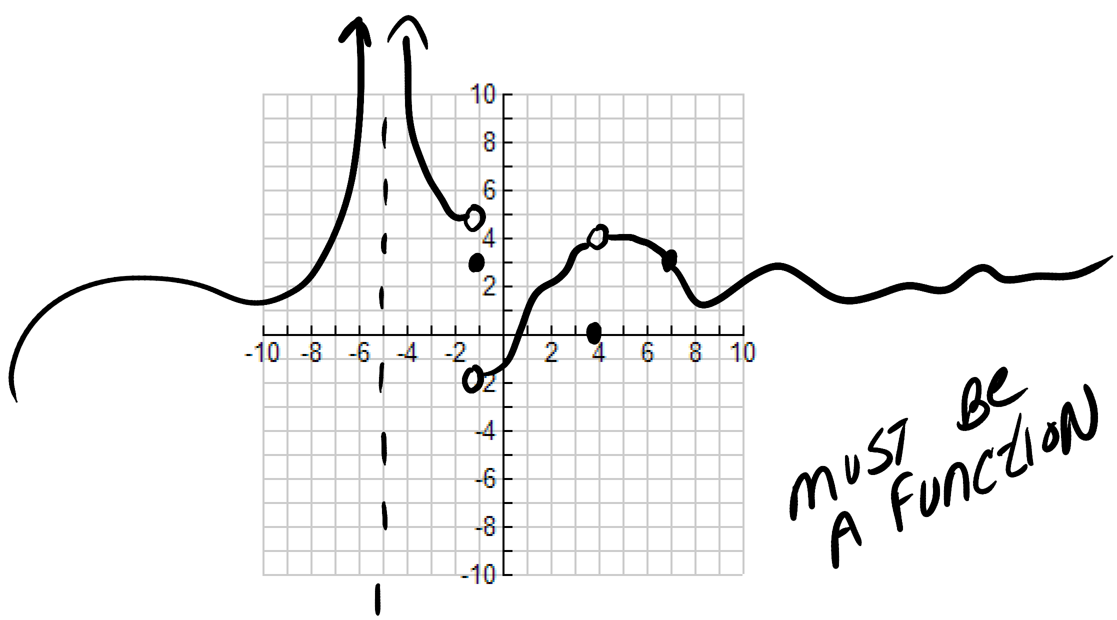
Limit from left is 5 but limit from right is - 2 AND point at (- 1, 3)

$\lim_{x \to 4} f(x) = 4$ $\qquad f(4) = 0$

Limit from left and right of x = 4 is 4 but point at (4, 0)

$\lim_{x \to 7} f(x) = 3$ $\qquad f(7) = 3$

Limit from left and right of 4 = 7 is 3 AND point at (7, 3)



FR2

Let $g(x)$ be the function $g(x)=\begin{cases}\sqrt[3]{2x} & for\, x\leq 4 \\ 6-x & for\, x>4\end{cases}$

Is $g(x)$ continuous for all real numbers? [You must prove with CALCULUS]

Need $g(4)=\lim\limits_{x\to 4} g(x)$

$g(4)=\sqrt[3]{8}$

$g(4)=2$

$\lim\limits_{x\to 4^-} g(x)=2$

$\lim\limits_{x\to 4^+} g(x)=2$ Hence, $\lim\limits_{x\to 4} g(x)=2$

Since $g(4)=\lim\limits_{x\to 4} g(x)$, then the function $g(x)$ is continuous at $x=4$ and continuous for $x\in$ reals.

Notes: Points were deducted for only having one-sided limits, talking about a function other than the given function, poor notation, and no concluding statement. Also, some students never addressed the function's value at x=4.

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FR#3

$f(x)$ and $g(x)$ are continuous functions with selected values given in the table below. The function $h(x) = f(g(x)) - 4$. Show that $h(x)$ must have at least one zero for the interval $0 < x < 6$. [USE CALCULUS]

x	$f(x)$	$g(x)$
0	11	2
2	3	6
4	1	4
6	2	0

$$h(0) = f(g(0)) - 4 = f(2) - 4 = -1$$

$$h(6) = f(g(6)) - 4 = f(0) - 4 = 7$$

By the I.V.T., there is an x, $0 < x < 6$, such that $h(0) < h(x) < h(6)$.

Since $h(0) = -1$ and $h(6) = 7$, then there is an x, $0 < x < 6$, such that $-1 < h(x) < 7$.

Hence, there must exist at least one $h(x) = 0$ for $0 < x < 6$.

Note: Once again, some students had problems with standard mathematical notation.