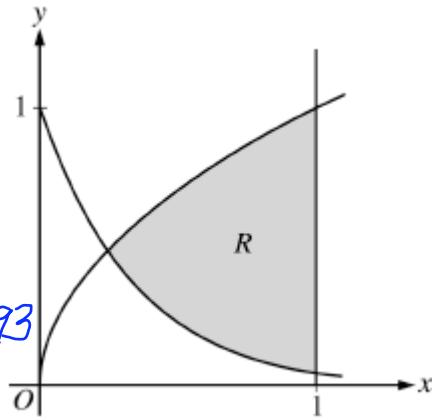


17 March 2010

Happy St. Patrick's Day

Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$ as shown below.

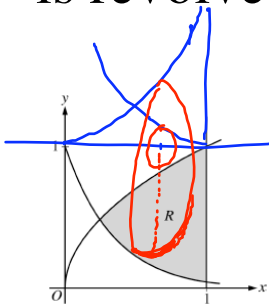
$$\begin{aligned} \sqrt{x} &= e^{-3x} \\ \text{at } x &= a \\ a &\approx .2387341293 \end{aligned}$$



(a) Find the area of R

$$\text{AREA}_R = \int_a^1 [\sqrt{x} - e^{-3x}] dx \approx .443 \text{ or } .442$$

(b) Find the volume of the solid generated when R is revolved around the horizontal line $y = 1$



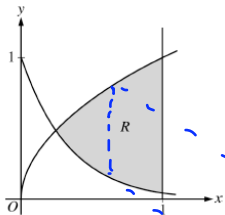
$$R(x) = 1 - e^{-3x}$$

$$r(x) = 1 - \sqrt{x}$$

$$V = \pi \int_a^1 [(1 - e^{-3x})^2 - (1 - \sqrt{x})^2] dx$$

$$V \approx .453\pi$$

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.



$$A(x) = bh$$

$$h = 5b$$

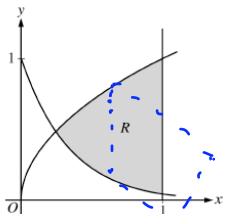
$$A(x) = 5b^2$$

$$b = \sqrt{x} - e^{-3x}$$

$$V = 5 \int_a^1 [\sqrt{x} - e^{-3x}]^2 dx$$

$$\approx 1.554$$

(d) The region R is now the base of another solid. For this solid, each cross section perpendicular to the x-axis are squares. Find the volume of this solid.



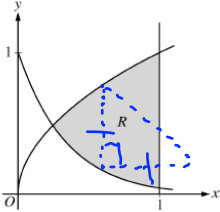
$$A(x) = s^2$$

$$s = \sqrt{x} - e^{-3x}$$

$$V = \int_a^1 [\sqrt{x} - e^{-3x}]^2 dx$$

$$\approx 0.3108$$

(e) The region R is now the base of another solid. For this solid, each cross section perpendicular to the x-axis are isosceles right triangles whose legs lie on the base. Find the volume of this solid.



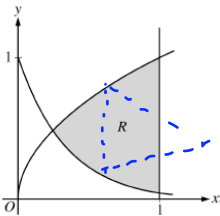
$$A = \frac{1}{2}bh \text{ or } \frac{1}{2}b^2$$

$$b = \sqrt{x} - e^{-3x}$$

$$V = \frac{1}{2} \int_a^1 [\sqrt{x} - e^{-3x}]^2 dx$$

$$= \frac{1}{2} (3.108)$$

(f) The region R is now the base of another solid. For this solid, each cross section perpendicular to the x-axis are equilateral triangles. Find the volume of this solid.



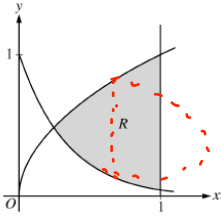
$$A(x) = \frac{\sqrt{3}}{4} b^2$$

$$b = \sqrt{x} - e^{-3x}$$

$$V = \frac{\sqrt{3}}{4} \int_a^1 [\sqrt{x} - e^{-3x}]^2 dx$$

$$= \frac{\sqrt{3}}{4} (3.108)$$

(g) The region R is now the base of another solid. For this solid, each cross section perpendicular to the x-axis are semi-circles. Find the volume of this solid.



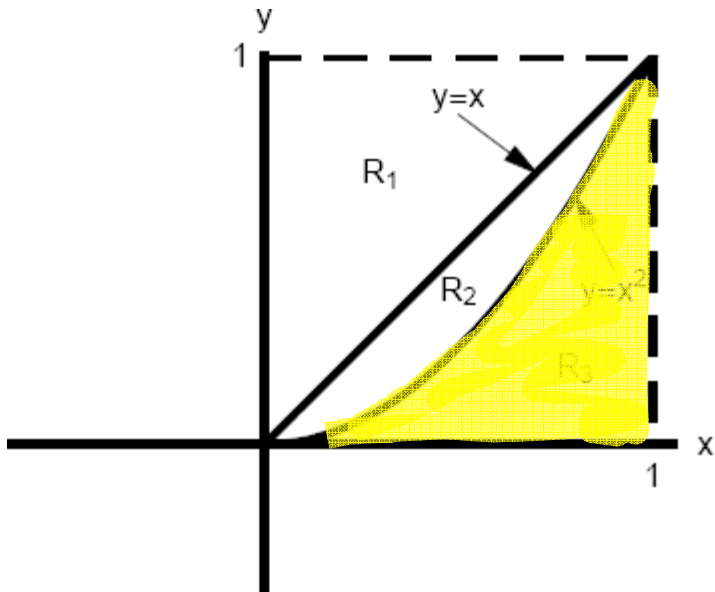
$$A(x) = \frac{\pi}{2} r^2$$

$$r = \frac{\sqrt{x} - e^{-3x}}{2}$$

$$V = \frac{\pi}{2} \int_a^1 \left[\frac{\sqrt{x} - e^{-3x}}{2} \right]^2 dx$$

$$= \frac{\pi}{8} [.3108]$$

Return to Mr. Zab's volume handout



Let R_3 be the base of a region. If the cross sections of the solid perpendicular to the x -axis are squares

$$A(x) = s^2 \quad s = x^2$$

$$V = \int_0^1 (x^2)^2 dx$$

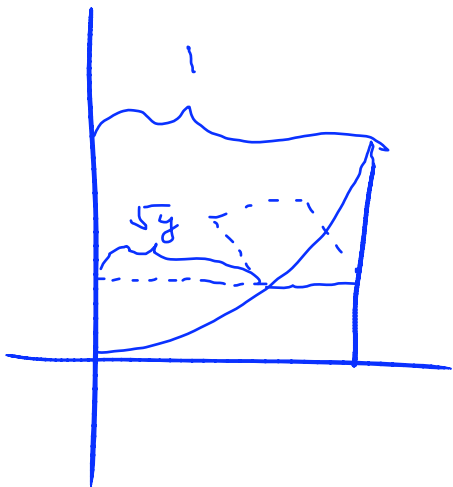
$$= \frac{x^5}{5} \Big|_0^1 = \frac{1}{5}$$

Let R_3 be the base of a region. If the cross sections of the solid perpendicular to the y -axis are squares.

$$A(y) = s^2$$

$$s = 1 - \sqrt{y}$$

BE CAREFUL!



$$V = \int_0^1 [1 - \sqrt{y}]^2 dy$$

$$\approx \frac{1}{6}$$