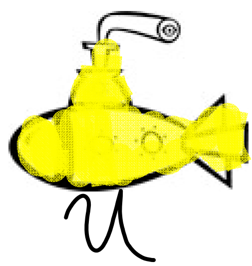
Integration by Substitution [Anti-Chain Rule]

$$\frac{d}{dx}[f(g(x))] = f'(g(x))\,g'(x)$$

$$\int f'(g(x))\,g'(x)\,dx = f(g(x)) + C$$

LOOK FOR A PRODUCT THAT YOU CAN'T EASILY SIMPLIFY

You should look for a composite function! Just like the Chain Rule, we'll need to find a u.



This is the u-sub!

Composite Function	**What is u?**	**What is $\frac{du}{dx}$?**
$\underline{2}\sin(\underbrace{2x}_{u})$	$2x$	2
$(3x^7+11)^5(21x^6)$	$3x^7+11$	$21x^6$
$\sqrt{4x^5-5x}\,(20x^4-5)$	$4x^5-5x$	$20x^4-5$
$\tan^2 x\sec^2 x$	$\tan x$	$\sec^2 x$

Here is our first example:

$$\int \underbrace{\left(x^2+5\right)}_{u}{}^{10}\underbrace{(2x)dx}_{du}$$

What is the composite function? $\left(x^2+5\right)^{10}$

What is the "inside" function [$g(x)$]?

Call it u $\quad u=\left(x^2+5\right)$

Find du

$$\frac{du}{dx}=2x$$

$$du=2x\ dx$$

Rewrite our original integral using all of the u-substitutions

$\int \left(x^2+5\right)^{10}(2x)dx$ becomes $\int u^{10}\,du$

Now solve in terms of u

$$=\frac{u^{11}}{11}+C$$

Now rewrite solution in terms of x

$$\int \left(x^2+5\right)^{10}(2x)dx=\frac{\left(x^2+5\right)^{11}}{11}+C$$

Our second example:

$$u = 2x$$

$$\int 2\sin(\underset{u}{2x})\,dx \qquad\qquad \frac{du}{dx} = 2$$

$$du = 2\,dx$$

Rewrite integral as:

$$\int \sin u \; du$$

$= -\cos u + C$ Rewrite solution in terms of x

$= -\cos(2x) + C$

Another example:

$$\int \sqrt{4x^5 - 5x}\;(20x^4 - 5)\,dx \qquad\qquad u = 4x^5 - 5x$$

Rewrite integral in terms of u

$$\frac{du}{dx} = 20x^4 - 5$$

$$du = \left(20x^4 - 5\right)dx$$

$$\int u^{\frac{1}{2}}\; du \qquad = \frac{2}{3}u^{\frac{3}{2}} + C$$

Rewrite your solution in terms of x:

$$\int \sqrt{4x^5 - 5x}\;(20x^4 - 5)\,dx = \frac{2}{3}\left(4x^5 - 5x\right)^{\frac{3}{2}} + C$$

In general, integration by u-sub looks like:

$$\int (composite\ \ function)(another\ \ function*)\ dx$$

*The other function looks like the derivative of the "inside" of the composite function. Look for the composite function!

$u = 5x^2 + 7$

$\frac{du}{dx} = 10x$

$du = 10x\ dx$

Try:

$$\int \frac{10x}{(5x^2+7)^3}\ dx$$

$= \int (5x^2+7)^{-3}(10x)\,dx$

$= \int u^{-3}\,du = \frac{u^{-2}}{-2} + C$

$= \frac{(5x^2+7)^{-2}}{-2} + C$

$$\int \tan^2 x \sec^2 x\ dx$$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x\ dx$

$= \int u^2\,du$

$= \frac{u^3}{3} + C$

$= \frac{\tan^3 x}{3} + C$

What if we do not quite have du?

$$\int x^3\left(x^4+3\right)^2 dx \qquad\qquad u = x^4+3$$

$$\frac{du}{dx} = 4x^3$$

$$du = 4x^3\,dx$$

Hey! We only have x^3, not $4x^3$. No worries, we can "fix" our substitution:

$$\frac{du}{4} = x^3\,dx$$

Remember, it's called $u-$substitution so YOU need to make all of the substitutions for what you have in the integrand.

$$\int u^2\,\frac{du}{4}$$

$$=\frac{1}{4}\int u^2\,du$$

CONSTANT MULTIPLE RULE

$$=\frac{1}{4}\left(\frac{u^3}{3}\right)+C$$

Now rewrite solution in terms of x

$$\int x^3\left(x^4+3\right)^2 dx = \frac{1}{12}\left(x^4+3\right)^3+C$$

$$\int \cos(3x)\,dx$$

$$u = 3x$$

$$\frac{du}{dx} = 3$$

$$du = 3\,dx$$

$$\frac{du}{3} = dx$$

$$\frac{1}{3}du = dx$$

$$\frac{1}{3}\int \cos u\ du =$$

CONSTANT MULTIPLE RULE STRIKES AGAIN!

$$\frac{1}{3}\sin u + C$$

$$\int \cos(3x)\,dx = \frac{1}{3}\sin(3x) + C$$

Consider: $\int 2x^3\sqrt{x^4+5}\ dx$

$$u = x^4 + 5$$

$$du = 4x^3\,dx$$

$$\frac{1}{2}du = 2x^3\,dx$$

u-sub work $\quad \frac{1}{2}\int u^{\frac{1}{2}}\,du = \frac{1}{2}\left(\frac{2}{3}u^{\frac{3}{2}}\right) + C$

$$\int 2x^3\sqrt{x^4+5}\ \ dx = \frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}} + C$$

Try:

$\int x(x^2+3)^9\,dx$

$u = x^2+3$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int u^9\,du$

$= \frac{1}{2}\left(\frac{u^{10}}{10}\right)+C$

$= \frac{1}{20}(x^2+3)^{10}+C$

$\int \frac{x^5+x^3}{x^2}\,dx$ Do we need u-substitution? NO!

SIMPLIFY

$= \int (x^3+x)\,dx$

$\int \frac{3x^2}{(x^3+1)^4}\,dx$ Do we need u-substitution?

$u = x^3+1$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

$= \int u^{-4}\,du$

$\int \sin^2\theta\cos\theta\,d\theta$ Do we need u-substitution?

$u = \sin\theta$

$\frac{du}{d\theta} = \cos\theta$

$du = \cos\theta\,d\theta$

$= \int u^2\,du$

$\int \left(\sin^2\theta + \cos^2\theta\right) d\theta$ Do we need u-substitution? NO!

$= \int 1 \, d\theta$

$= \theta + C$

Here is a different looking one!

$\int \underline{5x^4} \, g'\left(x^5\right) \underline{dx}$

$u = x^5$

$\frac{du}{dx} = 5x^4$

$du = 5x^4 \, dx$

$= \int g'(u) \, du$

$= g(u) + C$

$= g(x^5) + C$

Homework: page 304 #7, 9, 13, 19, 21, 23

SHOW all STEPS

thank you