Using Tables in AP Calculus Problems

The following problems were taken from:

Reasoning from Tabular Data

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Let $y(t)$ represent the population of a town over a 20-year period, where y is a differentiable function of t. The table below shows the population recorded at selected times.

t (yrs)	0	4	10	13	20
$y(t)$ (people)	2500	2724	3108	3697	4283

1. Use data from the table to find an approximation for $y'(12)$, and explain the meaning of $y'(12)$ in terms of the population of the town. Show the computations that lead to your answer.

2. Use data from the table and a trapezoidal approximation with four subintervals to approximate the average population of the town over the 20-year period. Show the computations that lead to your answer.

Another problem:

The rate at which water flows into a tank, in gallons per hour, is given by a positive continuous function R of time t. The table below shows the rate at selected values of t for a 12-hour period.

t (hrs)	0	2	4	6	8	10	12
$R(t)$(gal/hr)	12.5	13.4	13.9	14.3	14.6	14.8	14.7

1. Use a midpoint Riemann sum with three subintervals to approximate:

$$\int_0^{12} R(t)dt,$$

and explain the meaning of this definite integral in terms of the water flow, using correct units. Show the computations that lead to your answer.

2. A model for the rate of water flow is given by the function:

$$P(t) = \frac{1}{60}\left(750 + 24t - t^2\right)$$

where the positive rate P is measured in gallons per hour and the time t is measured in hours. Use $P(t)$ to find the average rate of water flow during the 12-hour time period. Indicate units of measure.

How about a long problem?

Use your graphing calculator, and give decimal answers correct to three decimal places.

1. Let $y(t)$ represent the temperature of a pie that has been removed from a 450°F oven and left to cool in a room with a temperature of 72°F, where y is a differentiable function of t. The table below shows the temperature recorded every five minutes.

t (min)	0	5	10	15	20	25	30
$y(t)$ (°F)	450	388	338	292	257	226	200

a. Use data from the table to find an approximation for $y'(18)$, and explain the meaning of $y'(18)$ in terms of the temperature of the pie. Show the computations that lead to your answer, and indicate units of measure.

b. Use data from the table to find the value of $\int_{10}^{25} y'(t)\,dt$, and explain the meaning of $\int_{10}^{25} y'(t)\,dt$ in terms of the temperature of the pie. Indicate units of measure.

c. A model for the temperature of the pie is given by the function:

$$W(t) = 72 + 380e^{-0.036t},$$

where t is measured in minutes and $W(t)$ is measured in degrees Fahrenheit (°F). Use the model to find the value of $W'(18)$. Indicate units of measure.

d. Use the model given in part (c) to find the time at which the temperature of the pie is 300°F.

Problems based on previous released AP questions: [written by Ms. McCleary]

x	- 8	- 6	- 4	- 2	0	2	4	6	8
$f'(x)$	4	6	0	- 6	- 4	- 2	0	6	4

The derivative f' of a function f is continuous and has exactly two zeroes. Selected values of f' are given in the table above. The domain of f is the set of all real numbers. [Justify everything with Calculus!]

(a) Estimate $f''(5)$

(b) Find any interval(s) where f is decreasing

(c) Given that $f(0)=7$ find $f(2)$

(d) Show that $f'(x)=5$ at least twice on the interval [- 8, 8]

(e) Based on the table, how many points of inflection must the graph of f have.

x	*h(x)*
1	1
2	3
3	6
4	10
5	15

$h(x)$ **is a twice-differentiable function.**

Is it increasing and an increasing rate or increasing at a decreasing rate? What does this mean in terms of its first and second derivative?

x	*g(x)*
1	1
2	5
3	8
4	10
5	11

$g(x)$ **is a twice-differentiable function.**

Is it increasing and an increasing rate or increasing at a decreasing rate? What does this mean in terms of its first and second derivative?

And now for some multiple-choice questions from previously released exams:

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

A table of values for a continuous function f is shown above. If four equal subintervals of $[0,2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

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x	2	5	7	8
$f(x)$	10	30	40	20

5. The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table above. Using the subintervals $[2,5]$, $[5,7]$, and $[7,8]$, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

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