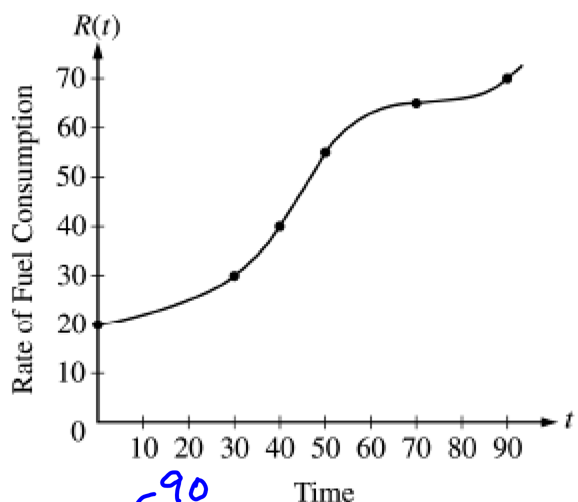
Trapezoid Rule-another geometric approach to definite integrals

-used when given data or used when we don't know the anti-derivative of the integrand
-similar to Rectangular Approximation Method [RAM] but uses trapezoids not rectangles [hence the name]
- intervals do NOT need to have same width

Let's revisit the airplane problem!

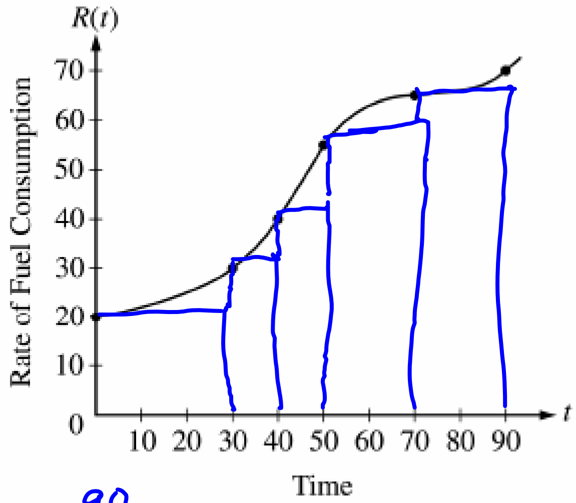
2003 AB 3 [calculator-friendly]



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

$\int_0^{90} R(t)\,dt \approx$ # of gallons used by plane during $0 \le t \le 90$ MINUTES

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

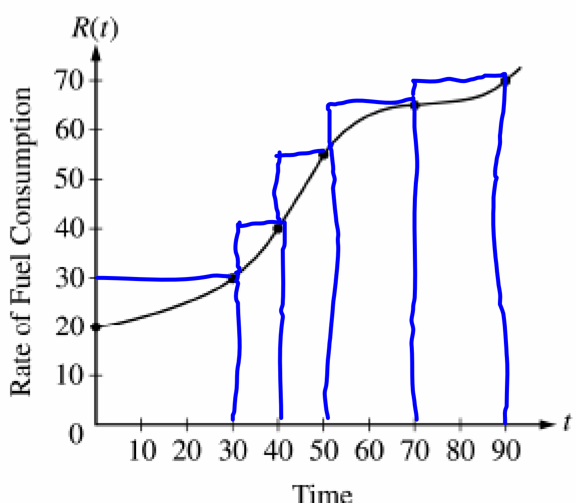


t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

$$\int_0^{90} R(t)\,dt \approx LRAM$$

$$LRAM = 30R(0) + 10R(30) + 10R(40) + 20R(50) + 20R(70)$$

$$= 3700 \text{ gallons}$$

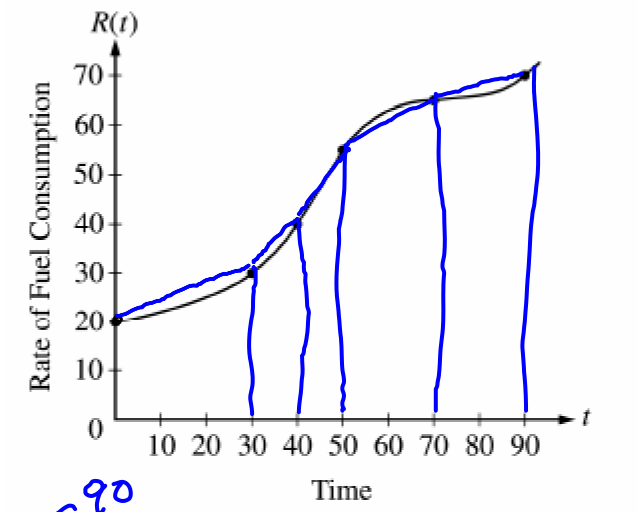


t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

$$\int_0^{90} R(t)\,dt \approx RRAM$$

$$RRAM = 30R(30) + 10R(40) + 10R(50) + 20R(70) + 20R(90)$$

$$= 4550 \text{ gallons}$$



bases

t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

Area of TRAPEZOID $= \frac{b_1+b_2}{2}h$

$$\int_0^{90} R(t)\,dt \approx TRAP$$

$$TRAP = \frac{R(0)+R(30)}{2}(30) + \frac{R(30)+R(40)}{2}(10) + \frac{R(40)+R(50)}{2}(10)$$

$$+ \frac{R(50)+R(70)}{2}(20) + \frac{R(70)+R(90)}{2}(20)$$

$$= 4125 \text{ gallons}$$

Is LRAM an over- or under-estimate? under

Is RRAM an over- or under-estimate? OVER

Is TRAP an over- or under-estimate? OVER

IF CURVE is CONCAVE UP, then TRAP is OVER

Which is the best estimate? TRAPEZOID

Why? FOLLOWS the CURVE BETTER

What do you obtain when you take the average of LRAM and RRAM? $\frac{3700+4550}{2} = 4125$ ← TRAPEZOID

ACK!

Example 2

Using the table, estimate the total distance traveled from time $t = 0$ to $t = 6$ using LRAM, RRAM, MRAM, and the Trapezoid rule

Time, t	0	1	2	3	4	5	6
Velocity, $v(t)$	3	4	5	4	7	8	11

Since $v(t) > 0$ on this interval, then the total distance traveled can be found by $\int_0^6 v(t)\,dt$

Now let's estimate using LRAM, RRAM, MRAM, and try using trapezoids. Notice that the intervals are evenly spaced.

$$\int_0^6 v(t)\,dt \approx LRAM$$

LRAM = $1\left[v(0)+v(1)+v(2)+v(3)+v(4)+v(5)\right]$

$= 31$

$$\int_0^6 v(t)\,dt \approx RRAM$$

RRAM = $1\left[v(1)+v(2)+v(3)+v(4)+v(5)+v(6)\right]$

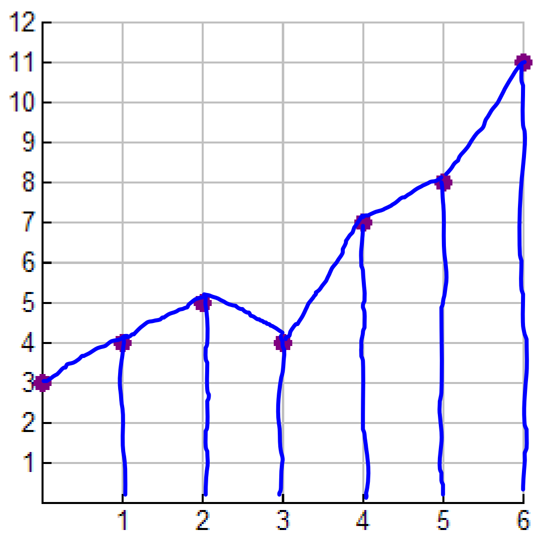
$= 39$

$$\int_0^6 v(t)dt \approx MRAM$$

MRAM= $2\left[v(1)+v(3)+v(5)\right]$

$= 32$

Let's draw in the trapezoids!



SIX TRAPEZOIDS AND they Have the Same height

$$\int_0^6 v(t)\,dt \approx TRAP$$

Let's sum up the areas of each trapezoid and look for a pattern.

$$TRAP = (1)\left(\frac{v(0)+v(1)}{2}\right)+(1)\left(\frac{v(1)+v(2)}{2}\right)+(1)\left(\frac{v(2)+v(3)}{2}\right)+$$
$$(1)\left(\frac{v(3)+v(4)}{2}\right)+(1)\left(\frac{v(4)+v(5)}{2}\right)+(1)\left(\frac{v(5)+v(6)}{2}\right)$$

Let's simplify this so that the computation is easier.

$$TRAP = \frac{1}{2}\left(v(0)+2v(1)+2v(2)+2v(3)+2v(4)+2v(5)+v(6)\right)$$

$$= 35$$

Hmm! Isn't 35 the average of LRAM and RRAM?
There is a formula that you memorize but it is probably just as easy to find the sum of the trapezoid areas. Here is the formula:

Let f be continuous on $[a,b]$. The Trapezoid Rule for approximating $\int_a^b f(x)\,dx$ is given by:

LEFT RIGHT

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left[f(x_0)+2f(x_1)+2f(x_2)+\ldots+2f(x_{n-1})+f(x_n)\right]$$

Where n is the number of trapezoids, a is the lower bound, and b is the upper bound. THE INTERVALS MUST BE EVENLY SPACED!!!
In our example above, $a = 0$, $b = 6$, $n = 6$

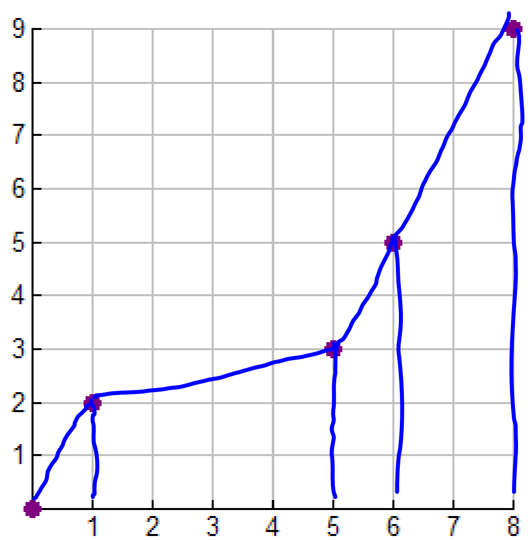
Always be very careful when given data in a table. If you do not have equal subdivisions, then the above formula does not work! If this is the case, then just find the sum of the areas of the trapezoids.

Consider the table of velocity values below and estimate the total distance traveled using the Trapezoid Rule. [Note: $v(t) \geq 0$ for the interval.

t (sec)	0	1	5	6	8
$v(t)$ $m/\sec$	0	2	3	5	9

$$TDT = \int_0^8 v(t)\,dt \approx TRAP$$

We cannot use the formula because we have unequal subdivisions.



$$TRAP = \frac{v(0)+v(1)}{2}(1) + \frac{v(1)+v(5)}{2}(4) + \frac{v(5)+v(6)}{2}(1) + \frac{v(6)+v(8)}{2}(2)$$

$$TRAP = (1)\left(\frac{0+2}{2}\right) + (4)\left(\frac{2+3}{2}\right) + (1)\left(\frac{3+5}{2}\right) + (2)\left(\frac{5+9}{2}\right)$$

$$TRAP = 1+10+4+14$$

$$TRAP = 29\,m$$

Let's re-visit the Heated Wire Problem [2005AB3]

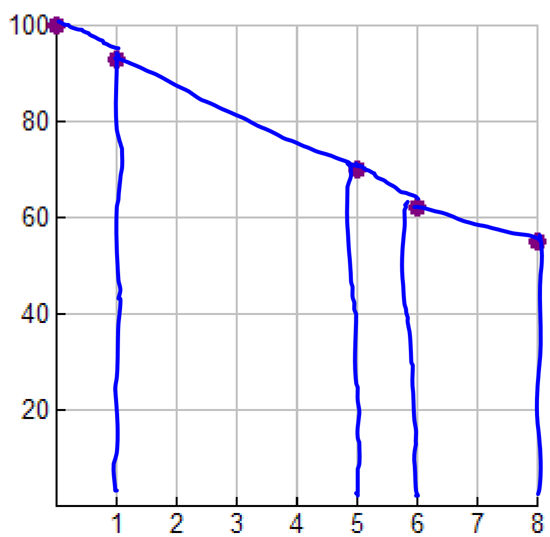
Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

$$\text{AV TEMP} = \frac{1}{8-0}\int_0^8 T(x)\,dx$$

$$\int_0^8 T(x)\,dx \approx TRAP$$



$$TRAP = \frac{T(0)+T(1)}{2}(1) + \frac{T(1)+T(5)}{2}(4) + \frac{T(5)+T(6)}{2}(1) + \frac{T(6)+T(8)}{2}(2)$$

$$= 605.5 \text{ cm}(°C)$$

$$\text{AV TEMP} = \frac{1}{8}(605.5)$$

$$= 75.6875\ °C$$

Try using the Trapezoid Rule for #48 on page 315

x	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
y	4.32	4.36	4.58	5.79	6.14	7.25	7.64	8.08	8.14

Use the Trapezoid Rule to estimate $\int_0^2 f(x)dx$

$a = 0$
$b = 2$
$n = 8$

$$TRAP = \frac{2-0}{2(8)}\left[4.32 + 2(4.36) + 2(4.58) + 2(5.79) + 2(6.14) + 2(7.25) + 2(7.64) + 2(8.08) + 8.14\right]$$

$$= \frac{1}{8}[100.14]$$

$$= 12.5175$$

Homework: page 315 #51, 52 AND page 320 10d
As always, show all steps