Sandy Point Beach Problem [2005 AB2 – calculator]


The tide removes sand from Sandy Point Beach at a rate modeled by the function *R*, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$$

A pumping station adds sand to the beach at a rate modeled by the function *S*, given by

$$S(t) = \frac{15t}{1+3t}$$

Both R(t) and S(t) have units of cubic yards per hour and t is measure in hours for $0 \le t \le 6$. At t = 0, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

$$\int_0^6 R(t)\,dt \approx 31.815$$
or $31.816\ yd^3$

(b) Write an expression for Y(t), the total number of cubic yards of sand on the beach at time t. [Be mindful of your variables!]

$$y(t) = 2500 + \int_0^t S(x)\,dx - \int_0^t R(x)\,dx$$

(c) Find the rate at which the total amount of sand is changing at time t = 4

$$y'(t) = \frac{d}{dt}\left[2500 + \int_0^t S(x)\,dx - \int_0^t R(x)\,dx\right]$$

$$y'(t) = S(t) - R(t)$$

$$y'(4) = S(4) - R(4)$$

$$y'(4) \approx -1.908 \text{ or } -1.909 \ yd^3/hr$$

(d) For $0 \leq t \leq 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

$$y'(t) = S(t) - R(t) = 0$$

let $S(t) = R(t)$ at $t \approx 5.117865$

$$y(0) = 2500 \ yd^3$$

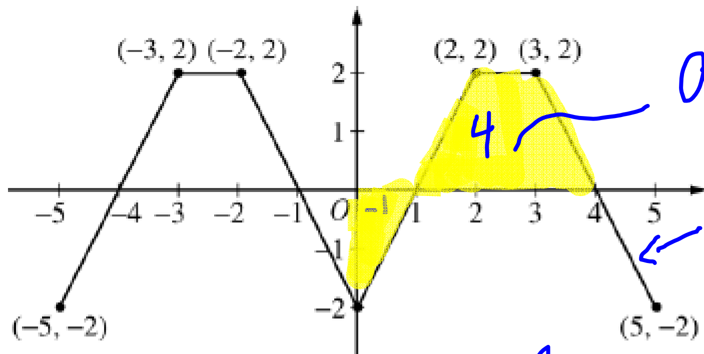
$$y(5.117865) = 2500 + \int_0^{5.117865} S(x)\,dx - \int_0^{5.117865} R(x)\,dx$$

$$\approx 2492.3694 \ yd^3$$

$$y(6) = 2500 + \int_0^6 [S(x) - R(x)]\,dx \approx 2493.2766 \ yd^3$$

at $x \approx 5.117$ or 5.118 the sand is at a minimum. The min. value is $2492.369\ yd^3$

Periodic Trapezoid Problem from 2010ftc.doc



$a = \frac{3+1}{2}(2)$

$m = \frac{0-2}{4-3}$

Graph of f and g'

The graph of f shown above consists of six line segments.

Let g be the function given by $g(x) = \int_0^x f(t)\,dt$

(a) **Find** $g(4)$, $g'(4)$, **and** $g''(4)$

$g(4) = \int_0^4 f(t)\,dt = -1 + 4 = 3$

$g'(x) = \frac{d}{dx}\int_0^x f(t)\,dt = f(x)$

$g'(4) = f(4) = 0$

$g''(x) = f'(x) \qquad g''(4) = f'(4) = -2$

(b) **Does g have a relative minimum, a relative maximum, or neither at x = 1. Justify**

At $x=1$ $g'(x) = f(x)$ changes from negative to positive values. ~~Hence~~ g has a rel min at $x=1$

(c) Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f. Given that g(5)=2, find g(10) and write an equation for the line tangent to the graph of g at x=108.

Do: The Water Tank Problem [2000 AB4], ~~Thomasville, Oregon Problem parts a and c only [2006 AB2]~~, and 2008AB4 B

the circle 2 problems