

[non-calculator problems]

How do problems #1 and #2 differ from each other?

1. The polynomial function  $f$  has selected values for its second derivative  $f''$  given in the table below. Based on the table, which of the following statements must be true?

$x$	0	1	2	3
$f''$	5	0	-7	4

- (A)  $f$  is increasing on the interval  $(0, 2)$
- (B)  $f$  is decreasing on the interval  $(0, 2)$
- (C)  $f$  has a local maximum at  $x = 1$
- (D) The graph of  $f$  has a point of inflection at  $x = 1$
- (E)** The graph of  $f$  changes concavity in the interval  $(0, 2)$

2. The function  $f$  has selected values for its second derivative  $f''$  given in the table below.  $f''$  is a strictly decreasing function for the interval  $[0, 3]$ . Based on the table, which of the following statements must be true?

$x$	0	1	2	3
$f''$	5	0	-3	-4

- (A)  $f$  is increasing on the interval  $(0, 2)$
- (B)  $f$  is decreasing on the interval  $(0, 2)$
- (C)  $f$  has a local maximum at  $x = 1$
- (D)** The graph of  $f$  has a point of inflection at  $x = 1$
- (E)** The graph of  $f$  changes concavity in the interval  $(0, 2)$

*at  $x=1$   $f''$  changes from positive to negative values*

How do problems #3 and #4 differ from each other?

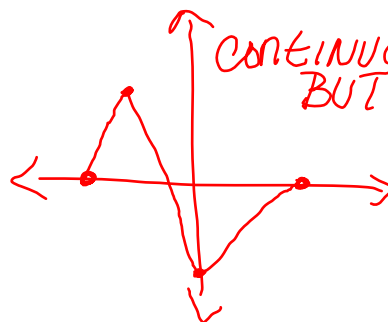
3. The function  $f$  is continuous for  $-2 \leq x \leq 2$  with selected values given in the table below. Based on the table, which of the following statements must be true?

$x$	-2	-1	0	2
$f(x)$	0	5	-5	0

- (A) There is some  $c, -2 < c < 2$  such that  $f'(c) = 0$
- (B) There is some  $c, -2 < c < 2$  such that  $f(c) = 10$
- (C) There is some  $c, -2 < c < 2$  such that  $f(c) = -10$
- (D)** There is some  $c, -1 < c < 0$  such that  $f(c) = 0$
- (E) There is some  $c, -2 < c < 2$  such that  $f''(c) = 0$

*← this is MVT*

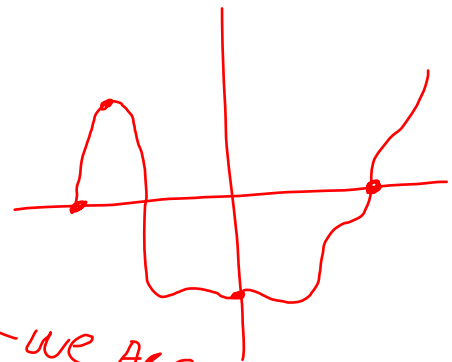
*IVT*



*CONTINUOUS BUT NOT DIFF.*

4. The function  $f$  is differentiable for  $-2 \leq x \leq 2$  with selected values given in the table below. Based on the table, which of the following statements must be true?

$x$	-2	-1	0	2
$f(x)$	0	5	-5	0



- (A) There is some  $c, -2 < c < 2$  such that  $f'(c) = 0$
- (B) There is some  $c, -2 < c < 2$  such that  $f(c) = 10$
- (C) There is some  $c, -2 < c < 2$  such that  $f(c) = -10$
- (D) There is some  $c, -1 < c < 0$  such that  $f'(c) = 0$
- (E) There is some  $c, -1 < c < 0$  such that  $f''(c) = 0$

MVT

WE ARE GUARANTEED  
 There is a  $c, -1 < c < 0$   
 such that  $f'(c) = \frac{f(0) - f(-1)}{0 - (-1)}$

How can you easily get "tricked" by problem #5?

5. Let  $f$  be a twice-differentiable function whose second derivative,  $f''$  is defined by

$$f''(x) = x(x+3)(x-3)^2. \text{ Where does } f \text{ have point(s) of inflection?}$$

- (A) At  $x=0$  only
- (B) At  $x=-3$  only
- (C) At  $x=0$  and  $x=-3$  only
- (D) At  $x=0, x=-3,$  and  $x=3$
- (E) At  $x=0$  and  $x=3$  only

POSSIBLE POINTS OF INFLECTION

at  $x=0, -3, 3$

$$\begin{aligned} (-\infty, -3) & f'' > 0 \\ (-3, 0) & f'' < 0 \\ (0, 3) & f'' > 0 \\ (3, \infty) & f'' > 0 \end{aligned}$$

$x=-3$   
 $x=0$  } Points  
 OF  
 INFLECTION  
 HERE  
 ONLY