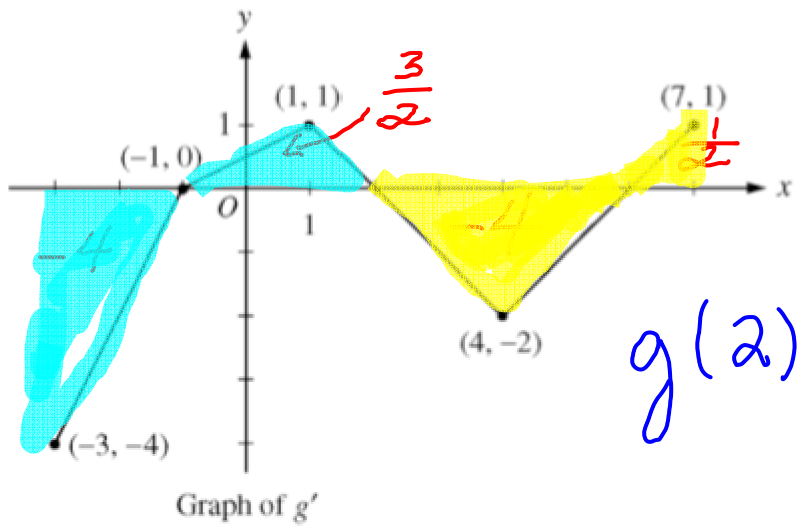
MORE CHAPTER 4 REVIEW



$g(2)=5$

Let g be a continuous function with g(2)=5. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$

At $x=2$, g' changes from positive to negative values. Hence g has a rel max at $x=2$ $\quad g(2)=5$

$$g(7) = g(2) + \int_2^7 g'(x)\,dx$$ ← area between $2 \le x \le 7$

$$= g(2) + g(x)\Big|_2^7$$

$$= g(2) + [g(7) - g(2)]$$

$$= 5 + -4 + \frac{1}{2} = \frac{3}{2}$$

$$g(-3) = g(2) + \int_{2}^{-3} g'(x)\,dx$$

$$= g(2) + -\int_{-3}^{2} g'(x)\,dx$$

$$= 5 - \left[-4 + \frac{3}{2}\right]$$

$$= 7.5$$ g has an ab max at $x = -3$
the ab max value is 7.5

(c) Find the average rate of change of g(x) on the interval $-3 \le x \le 7$

Av Rate of Δ on $[-3, 7]$ $= \dfrac{g(7) - g(-3)}{7 - (-3)}$

$$= \frac{1.5 - 7.5}{10}$$

$$= -.6$$

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

$a(t) = v'(t)$

av acc on $[0, 80]$ $= \dfrac{v(80) - v(0)}{80 - 0}$

$= \dfrac{49 - 5}{80} \dfrac{ft}{sec^2}$

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

$\int_{10}^{70} v(t)\,dt \approx MRAM$

$MRAM = 20\,[v(20) + v(40) + v(60)]$

$= 20\,[101]$

$= 2020$ ft

Because $v(t) > 0$ on $10 \le t \le 70$, then $\int_{10}^{70} v(t)\,dt$ gives us the TOTAL DISTANCE TRAVELED by the rocket on $10 \le t \le 70$ sec

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	−4	4	2
4	−1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

(c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)\,dt$. Find the value of $w'(3)$.

$$w(x) = \int_1^{g(x)} f(t)\,dt$$

$$w'(x) = \frac{d}{dx}\int_1^{g(x)} f(t)\,dt$$

use 2nd FTC

$$w'(x) = [f(g(x))][g'(x)]$$

$$w'(3) = f(g(3))g'(3)$$

$$= f(4)\,g'(3)$$

$$= (-1)(2)$$

$$w'(3) = -2$$