The exponential function e^x

If $f(x) = \ln x$, then $f^{-1}(x) = e^x$

Some rules to recall:

$$\ln e^x = x \ln e$$
$$= x$$

$$(e^x)^2 = e^{2x}$$

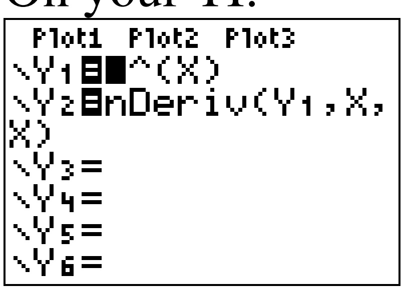
$$e^{\ln x} = x$$
$$e^a \bullet e^b = e^{a+b}$$
$$\frac{e^a}{e^b} = e^{a-b}$$
$$e^0 = 1$$

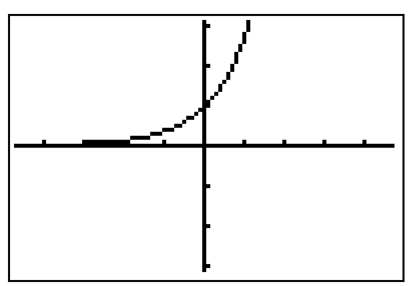
For $y = e^x$: Domain: $(-\infty, \infty)$ and Range : $(0, \infty)$

$$\lim_{x \to \infty} e^x = \infty$$

$\lim_{x \to -\infty} e^x = 0$ horizontal asymptote $y=0$ ♪ $e^x > 0$

On your TI:





$$\frac{d}{dx} e^x = e^x$$ It's a miracle!

Now that we know that $\frac{d}{dx}e^x = e^x$ we can analyze the graph of $f(x) = e^x$!

$f(x) = e^x$

$f'(x) = e^x$

Are there any critical values? No

Conclusion: $f'(x) > 0$ for $y' = e^x$

Then $f(x) = e^x$ is strictly increasing

$f''(x) = e^x$

Are there any possible points of inflection? No

Conclusion: $f''(x) > 0$ for $y'' = e^x$

Then $f(x)$ is always concave up

All of the rules of derivatives and integration still hold.

Here is a four star rule: ✵✵✵✵ $\frac{d}{dx}e^u = e^u \frac{du}{dx}$

u is a differentiable function of x

Solving equations with e^x or $\ln x$

1. $e^{\ln 2x} = 12$ Simplify!

$2x = 12$ Now solve.

$x = 6$ Feel free to check

2. $4e^{x} = 12$ Isolate

$e^{x} = 3$ Take ln of both sides

$\ln e^{x} = \ln 3$ Simplify and solve

$x = \ln 3$

3. $\ln x = 2$ Exponentiate both sides

$e^{\ln x} = e^{2}$ Simplify and solve

$x = e^{2}$

$\ln e^{2}$

$= 2\ln e$ ✓

4. $\ln(x-3) = 2$ Exponentiate both sides

$e^{\ln(x-3)} = e^{2}$ Simplify and solve

$x - 3 = e^{2}$

$x = e^{2} + 3$

$\ln(e^{2}+3-3)$

$= \ln e^{2}$ ✓

5. $\ln\sqrt{x+2} = 1$ Exponentiate both sides

$e^{\ln\sqrt{x+2}} = e^{1}$ Simplify

$\sqrt{x+2} = e$ Solve

$x+2 = e^{2}$

$x = e^{2} - 2$

$\ln\sqrt{e^{2}-2+2}$

$= \ln\sqrt{e^{2}} = \ln e = 1$ ✓

Now let's consider some Calculus using our new friend, e^x !

$\frac{d}{dx}\left(x^2 e^x\right)$ What rule? PRODUCT

$= 2xe^x + x^2e^x$

$= xe^x(2+x)$

$\frac{d}{dx}\left(\frac{x^2}{e^x}\right)$ What rule? Quotient

$= \frac{e^x(2x) - x^2e^x}{(e^x)^2}$

$= \frac{xe^x(2-x)}{e^{2x}}$

$\frac{d}{dx}\left(e^{5x}\right)$ What rule? CHAIN

Let's use our new four star rule: ✵✵✵✵ $\frac{d}{dx}e^u = e^u \frac{du}{dx}$

$\frac{d}{dx}e^{5x}$ $u=5x$

$\left[\text{think } \frac{d}{dx}e^u = e^u\frac{du}{dx}\right]$ $\frac{du}{dx}=5$

$= 5e^{5x}$

$\frac{d}{dx}\int_2^x e^t \, dt$ 2nd FTC again!

$= e^x$

$$\frac{d}{dx}\int_2^{x^2} e^t\,dt$$

$= e^{x^2}\left[\frac{d}{dx}x^2\right]$

$= 2x e^{x^2}$

$$\frac{d}{dx}e^{-x}$$

think $\frac{d}{dx}e^u$ $= e^u \frac{du}{dx}$

$u = -x$

$\frac{du}{dx} = -1$

$= -e^{-x}$

e^{-x}

$= \frac{1}{e^x}$

Another ❋❋❋❋ rule! $\int e^x\,dx = e^x + C$

Find the average rate of change of $f(x) = e^x$ on [0, 1]

a rate of Δ on $[0, 1]$ $= \frac{f(1) - f(0)}{1 - 0}$

$= e - 1$

Now find the average value of $f(x) = e^x$ on [0, 1]

$\frac{1}{1-0}\int_0^1 e^x\,dx$

$= e^x\Big|_0^1$

$= e - 1$

Try not to get fooled by functions involving $\ln x$ and e^x

$$\frac{d}{dx}\left(\ln\left(e^{x^2}\right)\right)$$ Take a deep breath and simplify first.

$$= \frac{d}{dx} x^2$$

$$= 2x$$

And now a little implicit differentiation to round out our e^x knowledge:

$$\frac{d}{dx}e^y = e^y \frac{dy}{dx}$$

Find the equation of the line tangent to the curve $xe^y + ye^x = 1$ at the point (0, 1)

$$\frac{d}{dx}xe^y + \frac{d}{dx}ye^x = \frac{d}{dx}1$$

$$e^y + xe^y\frac{dy}{dx} + e^x\frac{dy}{dx} + ye^x = 0$$

$$\frac{dy}{dx}\left[xe^y + e^x\right] = -ye^x - e^y$$

$$\frac{dy}{dx} = \frac{-ye^x - e^y}{xe^y + e^x}$$

$$\left.\frac{dy}{dy}\right|_{(0,1)} = \frac{-1-e}{0+1}$$

equation of tangent line at (0,1)

$$y - 1 = (-1-e)(x-0)$$

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Homework: pages 356, 357, 358 # 33, 35, 37, 45, 47, 49, 53, 57, 69