Integrating with e^x

$\int e^x \, dx = e^x + C$

Another ✵✵✵✵ rule!

$\int e^u \, du = e^u + C$ where u is a differentiable function of x

$\int 2e^{2x} \, dx$ Let $u = 2x$, $du = 2dx$

Rewrite as:

$\int e^u du = e^u + C$

$= e^{2x} + C$

$\int 10e^{10x} \, dx$ Let $u = 10x$

$du = 10\,dx$

$= \int e^u \, du$

$= e^u + C$

$= e^{10x} + C$

Here's a cool one:

$$\int (\cos x)(e^{\sin x})\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$$= \int e^u\,du$$

$$= e^u + C$$

$$= e^{\sin x} + C$$

Slightly harder:

$$\int \frac{e^{-x}}{1+e^{-x}}\,dx$$

Let $u =$ $1 + e^{-x}$

$du = -e^{-x}\,dx$

$-du = e^{-x}\,dx$

$$= -\int \frac{1}{u}\,du$$

$$= -\ln|u| + C$$

$$= -\ln|1+e^{-x}| + C$$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$$

Let's rewrite this as:

$$= \int e^{\sqrt{x}} \left(x^{\frac{-1}{2}} \right) dx$$

Let $u = \sqrt{x} = x^{\frac{1}{2}}$

$du = \frac{1}{2} x^{-\frac{1}{2}} dx$

$2du = x^{-\frac{1}{2}} dx$

$$= 2\int e^{u}\, du$$

$$= 2e^{u} + C$$

$$= 2e^{\sqrt{x}} + C$$

This one only looks hard:

[♪ the division bar!] 

$$\int \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}\, dx$$

$u = e^{x} - e^{-x}$

$du = [e^{x} + e^{-x}]dx$

$$= \int \frac{1}{u}\, du$$

$$= \ln|u| + C$$

$$= \ln|e^{x} - e^{-x}| + C$$

$\int e^x \sqrt{1-e^x}\ \ dx$ [♪ the grouping symbol] 

$= -\int u^{\frac{1}{2}} du$

$u = 1-e^x$

$du = -e^x dx$

$-du = e^x dx$

$= -\dfrac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= -\frac{2}{3}(1-e^x)^{\frac{3}{2}} + C$

This one is not as obvious as the other e^u-type integrals:

$\int \dfrac{5-e^x}{e^{2x}} dx$ *Rewrite as*:

$= \int \dfrac{5}{e^{2x}}\ dx \ - \ \int \dfrac{e^x}{e^{2x}}\ dx$ *Rewrite some more*:

$= \int 5e^{-2x}\ dx - \int e^{-x}\ \ dx$ Now start looking for *u*

$\int 5e^{-2x} dx$

$u = -2x$

$du = -2dx$

$-\frac{1}{2}du = dx$

$= -\frac{5}{2}\int e^u du$

$= -\frac{5}{2}e^u + C$

$= -\frac{5}{2}e^{-2x} + C_1$

$\int e^{-x}\ dx$

$u = -x$

$du = -dx$

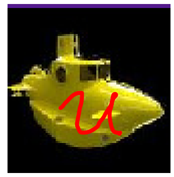
$-du = dx$

$= -\int e^u du$

$= -e^{-x} + C_2$

OUR SOLUTION $-\frac{5}{2}e^{-2x} + e^{-x} + C$

$\int e^{-x}\tan\left(e^{-x}\right)dx$



$u = e^{-x}$

$du = -e^{-x}\,dx$

$-du = e^{-x}\,dx$

$= -\int \tan u\,du$

FROM OUR PREVIOUS NOTES

$= \ln|\cos u| + C$

$= \ln|\cos e^{-x}| + C$

Definite integrals with e^x, e^u [Same as all the other definite integrals that we have done so far.]

$u = -2x$

$du = -2\,dx$

$-\frac{1}{2}\,du = dx$

$u(0) = 0$

$u(1) = -2$

$\int_0^1 e^{-2x}\,dx$

$= -\frac{1}{2}\int_0^{-2} e^u\,du$

$= \frac{1}{2}\int_{-2}^{0} e^u\,du$

$= \frac{1}{2} e^u \Big|_{-2}^{0}$

$= \frac{1}{2}\left[1 - e^{-2}\right]$

$$\int_0^1 x e^{-x^2}\, dx$$

$u = -x^2$

$du = -2x\, dx$

$-\frac{1}{2} du = x\,dx$

$u(0) = 0$

$u(1) = -1$

$$= -\frac{1}{2}\int_0^{-1} e^u\, du$$

$$= \frac{1}{2}\int_{-1}^{0} e^u\, du$$

$$= \frac{1}{2}\left[e^u \Big|_{-1}^{0}\right]$$

$$= \frac{1}{2}\left[1 - \frac{1}{e}\right]$$

What we learned today:

$\int e^x\, dx = e^x + C$ **and** $\int e^u\, du = e^u + C$

If it is not e^x, then it is e^u !!!!!

Homework: page 358 # 85, 86, 90, 100, 103, 104