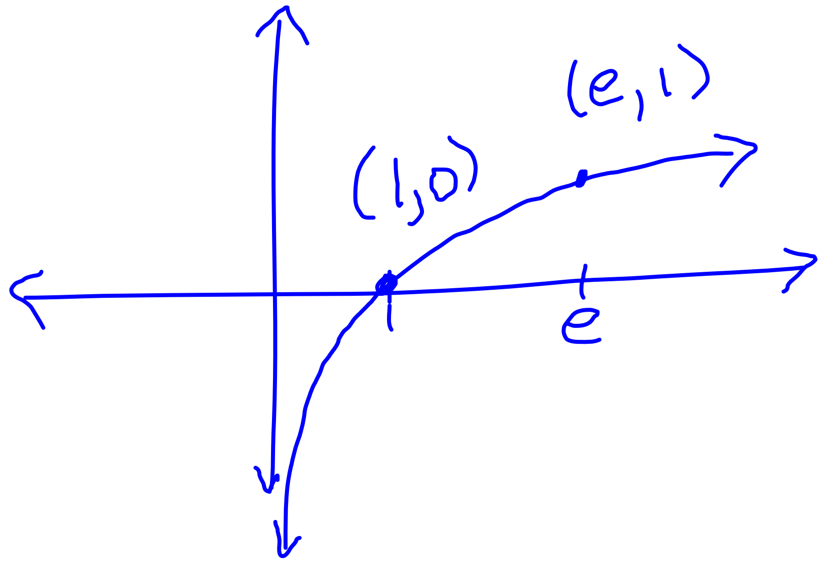
The Natural Log Function $f(x) = \ln x$

Stuff we [supposedly] already know:
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$



$\ln 1 = 0$

$\ln e = 1$

$\ln(ab) = \ln a + \ln b$

$\ln a^n = n \ln a$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

New Stuff:

$\lim_{x \to 0^+} \ln x = -\infty$

$\lim_{x \to \infty} \ln x = \infty$

$\frac{d}{dx} \ln x = \frac{1}{x}$ which means that $\int \frac{1}{x} dx = \ln x + C$

$\int x^{-1} dx = \ln x + C$

$\frac{d}{dx} \int_1^x \ln t \, dt = \ln x$

$\frac{d}{dx} \int_1^x \frac{1}{t} dt = \frac{1}{x}$

2nd FTC

Now that we know that $\frac{d}{dx}\ln x = \frac{1}{x}$ on $(0,\infty)$ we can analyze!!!

DOMAIN

Does $f(x) = \ln x$ have any critical values on its domain? Since zero is not in the domain, then there are no critical values!

On $(0,\infty)$, $f'(x) = \frac{1}{x}$ is always positive, therefore $f(x)$ is always increasing.

If $f(x) = \ln x$, then $f''(x) = \frac{-1}{x^2}$ and on $(0,\infty)$ $f''(x) < 0$, therefore $f(x)$ is always concave down. [No possible points of inflection.]

We now need to remember all of our Calculus from chapters one through four.

What is the average rate of change of $f(x) = \ln x$ on $[1, e]$?

AV RATE of Δ on $[1, e]$ $= \dfrac{f(e) - f(1)}{e - 1}$

$= \dfrac{1}{e-1}$

What is the average value of $f(x) = \ln x$ on $[1, e]$?

AV VALUE on $[1, e]$ $= \dfrac{1}{e-1}\int_1^e \ln x \, dx$

$= \dfrac{1}{e-1}(1)$

The Product Rule and the Quotient Rule along with the Chain Rule and u-sub still apply! Caution: you may want to simplify if you can before differentiating or integrating.

$\frac{d}{dx}\ln x^2$ [Can you simplify?] yes!

$= \frac{d}{dx}[2\ln x]$

$= 2\frac{d}{dx}[\ln x] = \frac{2}{x}$

$\frac{d}{dx}(x^2 \ln x)$ need PRODUCT RULE

$= 2x\ln x + \frac{1}{x}(x^2)$

$= 2x\ln x + x$

$\frac{d}{dx}\ln(2x)$ [Can you simplify?] yes

$= \frac{d}{dx}[\ln 2 + \ln x]$

$= \frac{1}{x}$

$\frac{d}{dx}\ln(5x)$ SAME thing

$= \frac{d}{dx}[\ln 5 + \ln x]$

$= \frac{1}{x}$

$n > 0$
$x > 0$

$\frac{d}{dx}[\ln(nx)]$

$= \frac{1}{x}$

$$\frac{d}{dx}\ln\left(x^2+1\right)$$

Need chain rule

$u = x^2+1$

$\frac{du}{dx} = 2x$

$= \frac{d}{dx} \ln u$

$= \frac{1}{u} \frac{du}{dx}$

$= \frac{1}{(x^2+1)}[2x]$

$$\frac{d}{dx}\ln\left(\frac{2}{x^2}\right)$$

[Can you simplify?] yes

$= \frac{d}{dx}\left[\ln 2 - \ln x^2\right]$

$= 0 - \frac{2}{x}$

Let $f(x) = \int_2^{\ln 2x} (t+1)\,dt$ and find $f'(x)$.

2nd FTC

$f'(x) = \frac{d}{dx}\int_2^{\ln 2x}(t+1)\,dt$

$= (\ln(2x)+1)\frac{d}{dx}\ln 2x$

$= \frac{\ln(2x)+1}{x}$

Finding equations of tangent lines – we still need the same two items: a point and a slope

Find the equation of the line tangent to the graph of $f(x)=3x^2-\ln x$ at $(1,3)$

$f'(x) = 6x - \frac{1}{x}$

$f'(1) = 6-1 = 5$

equation of tangent line at $(1,3)$

$y-3 = 5(x-1)$

Implicit Differentiation

$$\frac{d}{dx}\ln y=\frac{1}{y}\frac{dy}{dx}$$

Consider the curve: $x^2-3\ln y+y^2=0$ Find $\frac{dy}{dx}$

$\frac{d}{dx}x^2 - \frac{d}{dx}3\ln y + \frac{d}{dx}y^2 = \frac{d}{dx}0$

$2x - \frac{3}{y}\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}\left[-\frac{3}{y}+2y\right] = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y-\frac{3}{y}}$

SIMPLIFY FIRST

Consider the curve $\ln(xy)+5x=30$ Find $\frac{dy}{dx}$

$\ln x+\ln y+5x=30$

$\frac{d}{dx}\ln x+\frac{d}{dx}\ln y+\frac{d}{dx}5x=\frac{d}{dx}30$

$\frac{1}{x}+\frac{1}{y}\frac{dy}{dx}+5=0$

$\frac{dy}{dx}=\left[5-\frac{1}{x}\right]y$

You try:

$\frac{d}{dx}\ln x^4$ SIMPLIFY FIRST

$=\frac{d}{dx}\left[4\ln x\right]=\frac{4}{x}$

$\frac{d}{dx}\left(\underline{x^4}\,\underline{\ln x}\right)$ need PRODUCT RULE

$=4x^3\ln x+\frac{1}{x}(x^4)$

$\frac{d}{dx}\ln(\cos x)$ WOULD HAVE A RESTRICTED DOMAIN

need CHAIN RULE $\cos x>0$

$=\frac{d}{dx}\ln u$

$=\frac{1}{u}\frac{du}{dx}$

$=\frac{1}{\cos x}\left[-\sin x\right]$

$=-\tan x$

$u=\cos x$

$\frac{du}{dx}=-\sin x$

$$\frac{d}{dx}\left[\frac{\ln x}{x^4}\right]$$ need QUOTIENT RULE

$$= \frac{x^4\left(\frac{1}{x}\right) - (\ln x)(4x^3)}{(x^4)^2}$$

$$\frac{d}{dx}\sqrt{\ln x}$$ need CHAIN RULE

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$$= \frac{d}{dx} u^{\frac{1}{2}}$$

$$= \frac{1}{2} u^{-\frac{1}{2}} \frac{du}{dx}$$

$$= \frac{1}{2x\sqrt{\ln x}}$$

$$\frac{d}{dx}[\ln(2y)]$$ SIMPLIFY FIRST

$$= \frac{d}{dx}\left[\ln 2 + \ln y\right]$$

$$= \frac{1}{y}\frac{dy}{dx}$$

Homework: page 330 #46, 48, 54, 56, 64, 70, 80