Everything we need to know about FTC

Indefinite Integrals [or anti-differentiation]

$\int f'(x)\,dx = f(x) + C$ **where** $C \in$ **Reals**

What does this mean? It means that there are an infinite number of functions whose derivative is equal to $f'(x)$**.**
What do we need to remember about $f'(x)$**? We need to remember that every continuous function is some function's derivative.**

Here is an example:

$$\int (3x+7)\,dx = \frac{3x^2}{2} + 7x + C$$

Definite Integrals

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

We need to remember that $f'(x)$ **must be continuous on [a, b] and that the integrand,** $f'(x)$**, is a rate of change.**

Here is an example:

$$\int_1^5 (2x+5)\,dx = x^2 + 5x \Big|_1^5$$

$$= \left(5^2 + 5(5)\right) - \left(1^2 + 5(1)\right)$$
$$= 44$$

Properties of Integrals

If $f(x)$ **and** $g(x)$ **are continuous on [a, b], then**

(a) $\int_a^a f(x)\, dx = 0$

(b) **For** $c \in$ **Reals,** $\int_a^b c\, f(x)\, dx = c \int_a^b f(x)\, dx$

(c) $\int_b^a f(x)\, dx = -\int_a^b f(x)\, dx$

(d) **If** $a < c < b$**, then** $\int_a^c f(x)\, dx + \int_c^b f(x)\, dx = \int_a^c f(x)\, dx$

(e) $\int_a^b [f(x) \pm g(x)]\, dx = \int_a^b f(x)\, dx \pm \int_a^b g(x)\, dx$

(f) **If** $f(x)$ **is an odd function, then** $\int_{-a}^{a} f(x)\, dx = 0$

(g) **If** $f(x)$ **is an even function, then** $\int_{-a}^{a} f(x)\, dx = 2\int_0^a f(x)\, dx$

FTC

FTC I: $\int_a^b f'(x)\,dx = f(b) - f(a)$ **Example on previous page**

FTC II: $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$

Here is an example:

$$\frac{d}{dx}\left[\int_\pi^x \cos t\,dt\right] = \cos x$$

U-substitution

$$\int f'(g(x))\,g'(x)\,dx = f(g(x)) + C$$

Here is an example:

$\int 3(3x+7)^{10}\,dx$ **Let** $u = 3x+7$

$$\frac{du}{dx} = 3$$

$$du = 3dx$$

Rewrite in terms of "u": $\int u^{10}\,du$

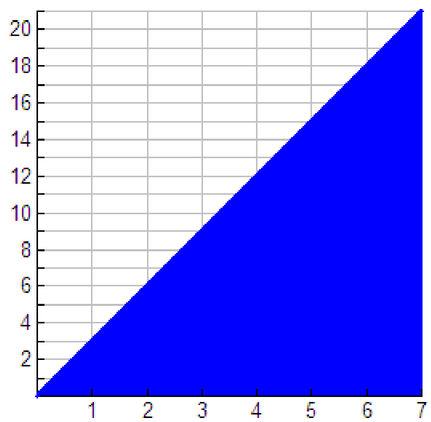
Solve: $\frac{u^{11}}{11} + C$

Rewrite in terms of "x" [our solution]: $\frac{(3x+7)^{11}}{11} + C$

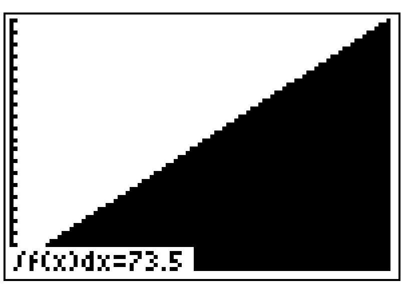
Thinking about integrals geometrically

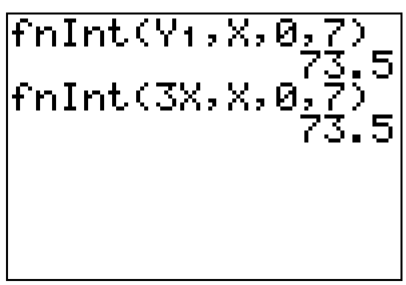
We first learned definite integrals as Riemann Sums, and then we learned to think of definite integrals as the "area between the curve and the x-axis".

$\int_0^7 3x\, dx$ **can be viewed as the blue area shown below**



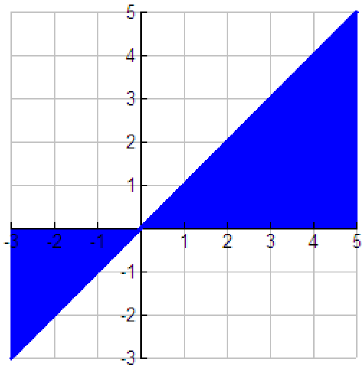
Remember that we can find definite integrals with our TI





We also learned to think about "signed area", meaning that any area below the x-axis needed to be considered negative [even though it drives some of us crazy!]

$\int_{-3}^{5} x\, dx$ can be viewed as



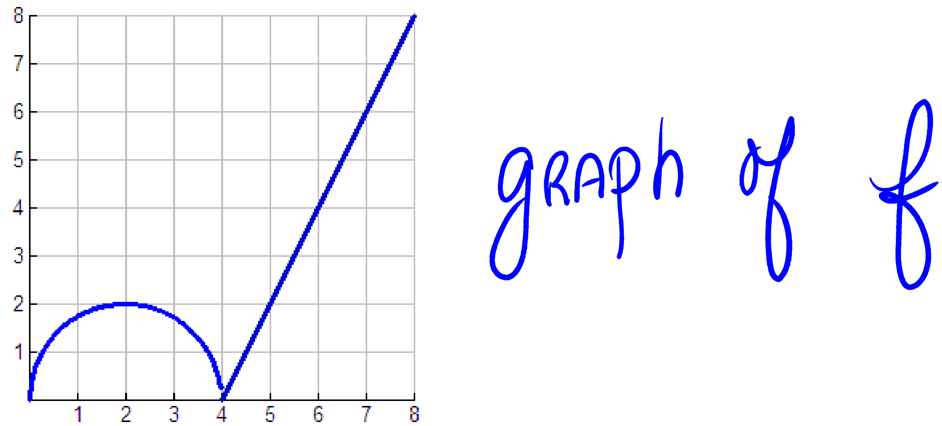
$\int_{-3}^{0} x\, dx = -4.5$ and $\int_{0}^{5} x\ dx = 12.5$ so $\int_{-3}^{5} x\, dx = -4.5 + 12.5$

$= 8$

Sometimes a function is defined as an integral. For example:

Let $F(x) = \int_{0}^{x} f\ (t)\ dt$

where the graph of f consists of a semi-circle and a line segment shown below



$$F(x) = \int_0^x f(t)\, dt$$

Find the following: $F(0),\ F(4),\ F(8)$

Our solutions:

$$F(0) = \int_0^0 f(t)\, dt = 0$$

$$F(4) = \int_0^4 f(t)\, dt$$

In order to find F(4) we just need to find the area of the semi-circle which will be

$F(4) = \frac{\pi}{2}\left(2^2\right)$ so, $F(4) = 2\pi$

$$F(8) = \int_0^8 f(t)\, dt$$

$$= \int_0^4 f(t)\, dt + \int_4^8 f(t)\, dt$$

$$= 2\pi + 16$$

This equals the area of the semi-circle plus the area of the triangle.

Thinking about definite integrals as accumulation functions

We know that the integral "accumulates" area between the curve and the x-axis [or the horizontal axis]. Another way to think of a definite integral is to think that we when we "accumulate" the rate of change, we will get the amount of change. Remember, the function in the integrand is some function's derivative.

The following definitions are from:
http://www.linmcmullin.net/PDF_Files/Rate_-_Accumulation.pdf

$$f(x) = f(a) + \int_a^x f(t)\ dt$$

$F(x) = F(x_0) + \int_{x_0}^{x} f(t)\,dt$ **where** F **is an anti-derivative of** f**,** x_0 **is a given initial value**

Remember: $f(t)$ **is a rate of change!!!!**
We did numerous problems from released AP exams that contained accumulation functions such as the "Sandy Point Beach" problem, the "Skeeter" problem, and numerous motion problems where we were given v(t) or a(t).