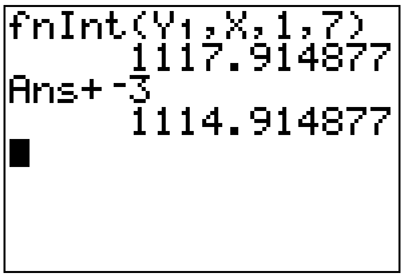
2010 Final exam solutions to the calculator section

51. A particle moves along the $x-$axis with velocity given by $v(t) = t + e^t$ for time $t \geq 0$.

If the particle is at position $x = -3$ at time $t = 1$, then what is the position of the particle at time $t = 7$? OR $x(1) = -3$

$$s(1) = -3$$

$$s(7) = s(1) + \int_1^7 v(t)\,dt$$



52. Let g be the function given by $g(x) = \int_1^x \left(100e^{-t^2}\right)\left(t^2 - 3t + 2\right)dt$

Which of the following statements about g must be true?

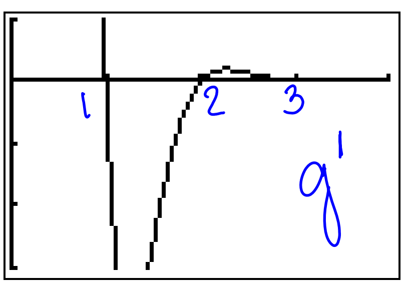
I g is increasing on (1, 2) NOPE $g' < 0$

II g is increasing on (2, 3) yes $g' > 0$

III $g(3) > 0$ NOPE

$$g'(x) = \frac{d}{dx}\int_1^x \left(100e^{-t^2}\right)\left(t^2 - 3t + 2\right)dt$$

$$g'(x) = \left(100e^{-x^2}\right)\left(x^2 - 3t + 2\right)$$



$$g(3) = \int_1^3 \left(100e^{-t^2}\right)\left(t^2 - 3t + 2\right)dt$$

$$g(3) < 0$$

See next page

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fnInt(Y1,X,1,3)
     -1.942140255
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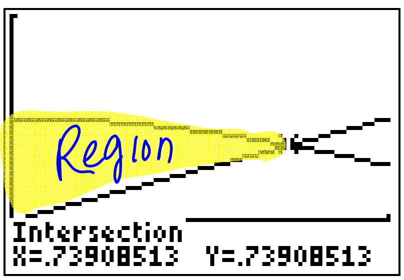
$g(3)$

53. Let $f(x)=x^3+x$. If h is the inverse function of f, then $h'(2)=$

$f(1)=2$ $\quad h=f^{-1}$ $\quad f'(x)=3x^2+1$

$h'(2)=\dfrac{1}{f'(1)}$ $\quad f'(1)=4$

54. The region in the first quadrant enclosed by the $y-$axis and the graphs of $y=\cos x$ and $y=x$ is rotated about the $x-$ axis. The volume of the solid generated is



$R(x)=\cos x$

$r(x)=x$

$$V=\pi\int_0^{.73908513}\left[(\cos x)^2-(x)^2\right]dx$$

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Ans→A
       .7390851332
fnInt(Y1²-Y2²,X,
0,A)
        .48389658
Ans*π
       1.520205941
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55. The average value of the function $f(x) = e^{-x^2}$ on the closed interval $[-1,1]$ is

$$\frac{1}{1-(-1)} \int_{-1}^{1} e^{-x^2}\,dx$$

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fnInt(e^(-X²),X,
-1,1)
          1.493648266
Ans/2
          .7468241328
█
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56. Let f be defined as follows, where $a \neq 0$

$$f(x) = \begin{cases} \dfrac{x^2 - a^2}{x-a} & \text{for } x \neq a \\ 2a & x = a \end{cases}$$

$$\lim_{x \to a} \frac{(x+a)\cancel{(x-a)}}{\cancel{x-a}} = 2a$$

$$f(a) = 2a$$

Which of the following are true about f?

I $\lim_{x \to a} f(x)$ exists ✓

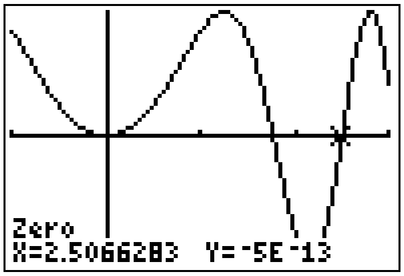
II $f(a)$ exists ✓

III $f(x)$ is continuous at $x = a$ ✓

57. If the function g is defined by $g(x) = \int_0^x \sin(t^2)\,dt$ on the closed interval $-1 \leq x\ 3$, then g has a local minimum at $x =$

$$g'(x) = \frac{d}{dx} \int_0^x \sin(t^2)\,dt = \sin(x^2)$$

LOOK FOR WHERE $g'(x)$ CHANGES FROM NEGATIVE TO POSITIVE VALUES



58.

Time (sec)	0	10	25	37	46	60
Rate (gal/sec)	500	400	350	280	200	180

The table above gives the values for the rate (in gallons/second) at which water flowed into a lake, with readings taken at specific times. A right Riemann sum, with the five subintervals indicated by the data in the table, is used to estimate the total amount of water that flowed into the lake during the time period $0 \le t \le 60$. What is this estimate?

$$\int_0^{60} R(t)\,dt \approx \text{RRAM}$$

$$\text{RRAM} = 10R(10) + 15R(25) + 12R(37) + 9R(46) + 14R(60)$$

59. If $G(x)$ is an anti-derivative for $f(x)$ and $G(3)=7$, then $G(10)=$

$$f(t) = G'$$

$$G(10) = G(3) + \int_3^{?} f(t)\,dt$$

$$= G(3) + G(10) - G(3)$$

60.

x	0	2	3	4
$f(x)$	e	10	25	π
$f'(x)$	1	3	7	15

The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval [0, 4], what is the value of $\int_0^4 f'(x)\,dx$

$$\int_0^4 f'(x)\,dx$$
$$= f(x)\Big|_0^4$$
$$= f(4) - f(0)$$
$$= \pi - e$$

61. The volume of an expanding sphere is increasing at a rate of 12 cubic feet per second. When the volume of the sphere is 36π cubic feet, how fast in square feet per second is the surface area increasing?

Note: $v = \dfrac{4\pi r^3}{3}$ and $S = 4\pi r^2$

$$\frac{d}{dt}S = \frac{d}{dt}4\pi r^2$$
$$\frac{dS}{dt} = 8\pi r\frac{dr}{dt}$$
$$\frac{dS}{dt} = 8\pi\cancel{(3)}\left(\frac{1}{\cancel{3\pi}}\right)$$
$$\frac{dS}{dt} = 8$$

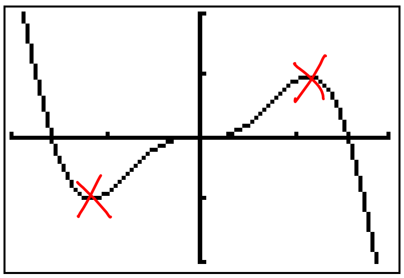
$$\frac{dV}{dt} = 12\ \frac{ft^3}{sec}$$
$$V = 36\pi\ ft^3$$
$$V = \frac{4\pi r^3}{3}$$
$$3\cancel{6\pi} = \frac{4\cancel{\pi} r^3}{3}$$
$$\frac{108}{4} = r^3$$
$$3ft = r$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$
$$12 = 4\pi(3^2)\frac{dr}{dt}$$
$$\frac{1}{3\pi} = \frac{dr}{dt}$$

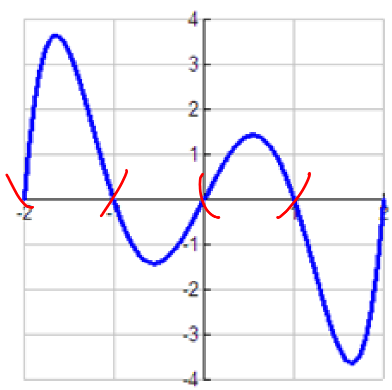
62. The function g is continuous for $-\pi \le x \le \pi$ and $g(-\pi) = g(\pi) = 0$. If there is no c where $-\pi < c < \pi$ for which $g'(c) = 0$, which of the following statements must be true?

MUST BE DIFFERENTIABLE FOR THE MVT TO APPLY

63. The derivative of the function $f'(x) = x^2 \sin(2x)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?



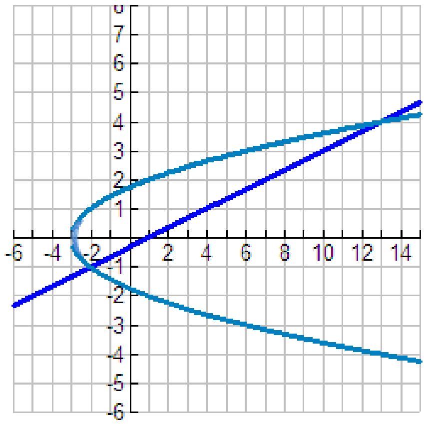
graph of $f'(x)$



Graph of f'

64. The graph of f', the derivative of f is show above for $-2 \le x \le 2$. On what intervals is f increasing?

LOOK FOR WHEN $f' > 0$



$y^2 - 3 = 3y + 1$

$y^2 - 3y - 4 = 0$

$(y-4)(y+1) = 0$

LEFT is $h(y)$
RIGHT IS $g(y)$

65.

Region R is bounded by $g(y) = y^2 - 3$ and $h(y) = 3y + 1$ as shown above. Which of the following expressions gives the area of Region R?

$$\text{Area}_R = \int_{-1}^{4} [h(y) - g(y)]\,dy$$

B4 [Mark as #66 on your Scantron]

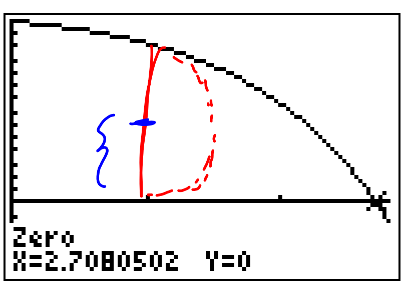
The base of a certain solid is the region in the first quadrant bounded by the $x-$axis and the $y-$axis and the curve $y = 15 - e^x$. If each cross section perpendicular to the $x-$axis is a semi-circle with diameter across the base, then the volume of this solid is

$0 = 15 - e^x$ at $x = A$

$A \approx 2.708050201$

$A(x) = \frac{\pi}{2} r^2$

$r = \frac{15 - e^x}{2}$



$$V = \pi \int_0^A \left[\frac{15 - e^x}{2}\right]^2 dx$$

If $\int_1^3 f(x)dx = k$ and $\int_1^7 f(x)dx = -4$, what is the value of $\int_7^3 [x + f(x)]dx$

Note: This is not a typo so don't get up!

$$\int_7^3 [x + f(x)]\,dx$$

$$\int_3^7 f(x)\,dx = \underbrace{\int_1^7 f(x)\,dx - \int_1^3 f(x)\,dx}_{= -4-k}$$

$$= -\int_3^7 [x + f(x)]\,dx$$

$$= -\left[\int_3^7 x\,dx + \int_3^7 f(x)\,dx\right]$$

$$= -\left[\frac{x^2}{2}\Big|_3^7 + (-4-k)\right]$$

$$= -\left[\frac{49}{2} - \frac{9}{2} - 4 - k\right]$$

$$= -[16 - k]$$

$$= -16 + k$$