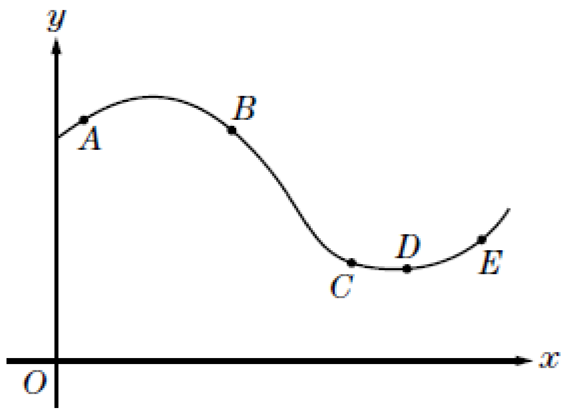
Solutions to the final exam of April 26



1.

At $x = B$, the graph of the function is both decreasing and concave down

2.

Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{4x}{y}$, where $y(2) = -2$?

$$\int y\,dy = \int 4x\,dx$$

$$\frac{y^2}{2} = 2x^2 + C$$

$$\frac{(-2)^2}{2} = 2(2^2) + C \qquad C = -6$$

$$\frac{y^2}{2} = 2x^2 - 6$$

$$y^2 = 4x^2 - 12$$

$$y = -\sqrt{4x^2 - 12}$$

$4x^2 - 12 \geq 0$

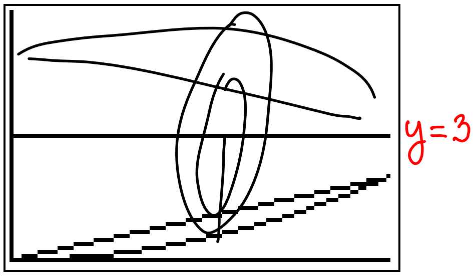
$4x^2 \geq 12$

$x^2 \geq 3$

$x > \sqrt{3}$

3.

Let S be the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$ for $0 \le x \le 1$. What is the volume of the solid generated when S is revolved about the line $y = 3$?



$$R(x) = 3 - 2x^2$$

$$r(x) = 3 - 2x$$

$$V = \pi \int_0^1 \left[(3-2x^2)^2 - (3-2x)^2 \right] dx$$

4.

If $y = 5 + \int_2^{2x} e^{-t^2}\, dt$, which of the following is true?

$$\frac{dy}{dx} = \frac{d}{dx}\left[5 + \int_2^{2x} e^{-t^2}\, dt\right]$$

$$= 2e^{-4x^2}$$

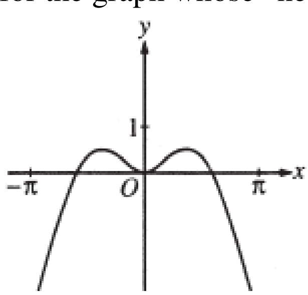
$$y(1) = 5 + \int_2^2 e^{-t^2}\, dt = 5$$

5.

The graphs of five functions are shown below. Which function has a nonzero average value over the closed interval $[-\pi, \pi]$?

Look for the graph whose “negative” area does NOT equal its “positive” area

(E)



6. If $f(x)=\sqrt{4\sin x+2}$, then $f'(0)=$

$f(x)= u^{\frac{1}{2}}$ where $u=4\sin x+2$

$f'(x)=\frac{1}{2\sqrt{u}}\frac{du}{dx}$ $\qquad \frac{du}{dx}=4\cos x$

$f'(x)=\frac{1}{2\sqrt{4\sin x+2}}\left(4\cos x\right)$

$f'(x)=\frac{4\cos 0}{2\sqrt{2}} = \frac{4}{2\sqrt{2}} = \sqrt{2}$

7. $\lim\limits_{x\to -1}\frac{x^2-3x-4}{x^2-1}=$

$\lim\limits_{x\to -1}\frac{\cancel{(x+1)}(x-4)}{\cancel{(x+1)}(x-1)}$

$=\lim\limits_{x\to -1}\frac{(x-4)}{(x-1)} = \frac{-5}{-2} = \frac{5}{2}$

8. If $\sin xy = x+y$, then $\frac{dy}{dx}=$

$\frac{d}{dx}\sin(xy) = \frac{d}{dx}x+\frac{d}{dx}y$

$\cos(xy)\left[y+x\frac{dy}{dx}\right] = 1+\frac{dy}{dx}$

$y\cos(xy)+\left[x\cos(xy)\right]\frac{dy}{dx} = 1+\frac{dy}{dx}$

$y\cos(xy)-1 = \frac{dy}{dx}\left[1-x\cos(xy)\right]$

$\frac{y\cos(xy)-1}{1-x\cos(xy)} = \frac{dy}{dx}$

9. $\int \frac{1}{16+x^2}\,dx =$

$a=4$

$u=x$

$du=dx$

$\int \frac{du}{a^2+x^2}$

$= \frac{1}{4}\arctan\left(\frac{x}{4}\right)+C$

10. Let f be a differentiable function such that $f(3)=7$, $f(10)=3$, $f'(3)=-3$ and $f'(10)=25$. The function g is differentiable and $g(x)=f^{-1}(x)$ for all x. What is the value of $g'(3)$?

$f(10)=3$ so $g(3)=10$

$g'(3)=\frac{1}{f'(10)}=\frac{1}{25}$

11. $\lim_{x\to-\infty}\frac{(x-3)(7-10x)}{(10+x)(25+x)}=$

use end behavior

$\lim_{x\to-\infty}\frac{-10x^2}{x^2}=-10$

12. $\int \frac{3}{7x^2}\,dx =$

$=\frac{3}{7}\int x^{-2}\,dx$

$=-\frac{3}{7}x^{-1}+C$

13. $\int [\sec^2(222x) + \sin(222x)]\, dx =$

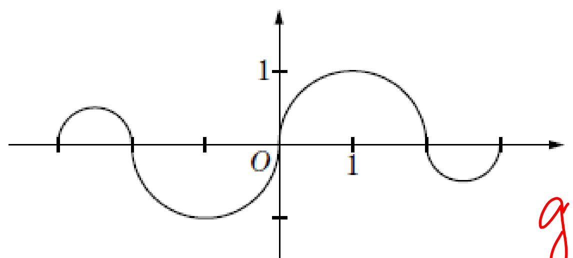
$u = 222x$

$du = 222\, dx$

$\frac{1}{222} du = dx$

$\frac{1}{222} \int [\sec^2 u + \sin u]\, du$

$= \frac{1}{222}\left[\tan(222x) - \cos(222x)\right] + C$



Graph of f

GRAPH of g'

$g'(x) = \frac{d}{dx} \int^{x} f(t)\, dt$

$g'(x) = f(x)$

14. The graph of the function f shown above consists of four semicircles. If $g(x) = \int_0^x f(t)\, dt$,

then which of the following statements is FALSE?

(A) $g(0) = 0$

(B) $g(x)$ is increasing on $[-3, -2]$ and $[0, 2]$ $g'(x) > 0$ ✓

(C) The graph of $g(x)$ has horizontal tangents at $x = -3$, $x = -2$, $x = 0$, $x = 2$ and $x = 3$ $g' = 0$ ✓

(D) The graph of $g(x)$ has a point of inflection at $x = 0$ NOPE!

(E) $g(2) > g(3)$

15.

$$\int_1^e \left(\frac{x^2-1}{x}\right)dx =$$

SIMPLIFY FIRST

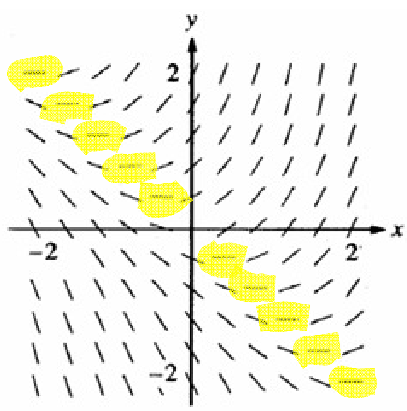
$$\int_1^e \left[x-\frac{1}{x}\right]dx$$

$$= \frac{x^2}{2} - \ln x \Big|_1^e$$

$$= \left(\frac{e^2}{2} - \ln e\right) - \left(\frac{1^2}{2} - \ln 1\right)$$

$$= \frac{e^2}{2} - 1 - \frac{1}{2}$$

16.



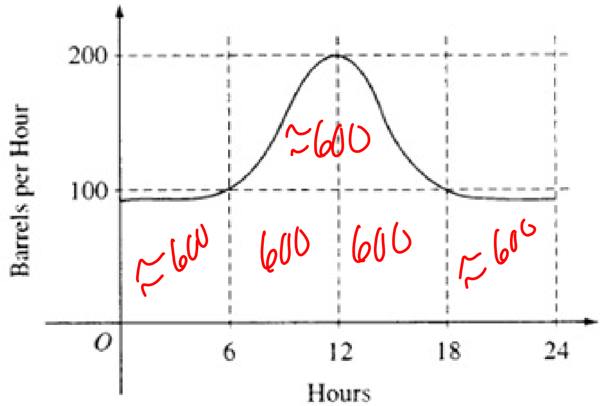
LOOK FOR horizontal TANGENTS

(1,-1), (-1,1), ...

Shown above is a slope field for which of the following differential equations?

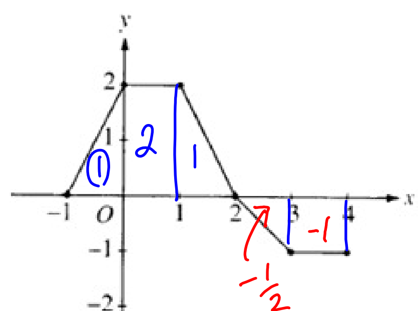
(A) $\frac{dy}{dx} = 1+x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x+y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

17.



The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day? $\approx 600(5)$

18.



. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

sum up the signed areas

19.

If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

(A) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$. MVT

(B) $f'(c) = 0$ for some c such that $a < c < b$. maybe

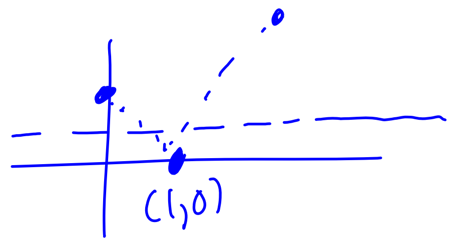
(C) f has a minimum value on $a \le x \le b$. EVT

(D) f has a maximum value on $a \le x \le b$. EVT

(E) $\int_a^b f(x)dx$ exists. yes

20.

x	0	1	2
$f(x)$	1	k	2



The function f is continuous on the closed interval $[0,2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0,2]$ if $k =$

INTERMEDIATE VALUE theorem

21. Let f be a function with a second derivative given by $f''(x) = x^4(x+10)(3-x)$. What are the $x-$coordinates of the points of inflection of the graph of f?

Possible P of I at $x=0, x=-10, x=3$

$(-\infty, -10)$ $f'' < 0$ — $x=-10$

$(-10, 0)$ $f'' > 0$

$(0, 3)$ $f'' > 0$ — $x=3$

$(3, \infty)$ $f'' < 0$

22. Let g be a function that is twice differentiable with $g(3)=7$, $g'(3)=-2$ and $g''(3)=5$.

What is the value of the approximation of $g(3.1)$ using the line tangent to the graph of g at $x=3$?

$y-7=-2(x-3)$

$y-7=-2(3.1-3)$

$y \quad = -0.2+7$

23. $\int \frac{x}{x^2+4}\,dx =$

$u=x^2+4$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln(x^2+4)+C$

24. Let $f(x)$ be the piece-wise function define as $f(x)=\begin{cases} \frac{x-3}{x^2-9} & \text{if } x \neq 3 \\ \frac{1}{6} & \text{if } x = 3 \end{cases}$

Which of the following statements about f are true?

I. f has a limit at $x=3$ ✓

II. f is not continuous at $x=-3$ ✓

III. f is continuous at $x=3$ ✓

$\lim_{x\to 3} \frac{\cancel{(x-3)}}{\cancel{(x-3)}(x+3)} = \frac{1}{6}$

$f(-3) = \frac{-6}{0}$ ACK!

If $f(x)=x^2e^x$, then the graph of f is decreasing for all x such that

$f'(x) = 2xe^x + x^2e^x$

at $x=0$

$x=-2$

$f'(x) = xe^x(2+x)$ C.U.

$(-\infty, -2)$ $f' > 0$ f increasing

$(-2, 0)$ $f' < 0$ f decreasing

$(0, \infty)$ $f' > 0$ f increasing

If $\int_a^b f(x)\,dx = a + 2b$, then $\int_a^b (f(x)+5)\,dx =$

$$\int_a^b f(x)\,dx + \int_a^b 5\,dx$$

$$= a + 2b + 5b - 5a$$

$\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\,dx$ is

$$\int_0^{\frac{\pi}{4}} e^{\tan x} \sec^2 x \, dx$$

$u = \tan x$

$du = \sec^2 x \, dx$

$u(0) = 0$

$u(\frac{\pi}{4}) = 1$

$$= \int_0^1 e^u \, du$$

$$= e^1 - e^0$$