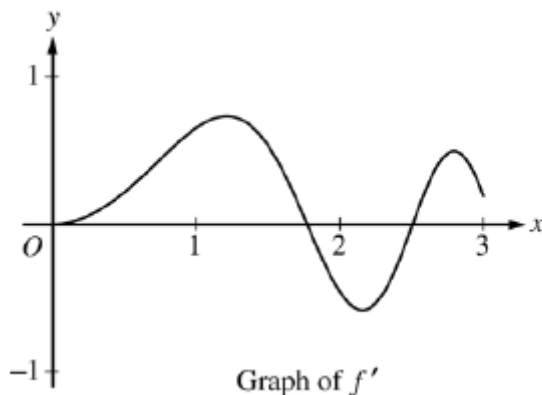


2006 AB2B [*calculator-friendly*]



Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f' , the first derivative of f , given by

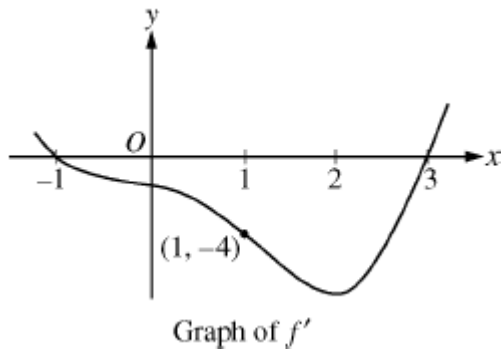
$f'(x) = e^{\frac{-x}{4}} \sin(x^2)$. The graph of $y = f'(x)$ is shown above.

(a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the open interval $1.7 < x < 1.9$. Justify.

(b) On the closed interval $0 \leq x \leq 3$, find the value of x at which f has an *absolute maximum*. Justify.

(c) Write an equation for line tangent to the graph of f at $x = 2$

2009 AB5B [non-calculator]



CLUE

Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f' , the derivative of f , is shown above. The graph of f' crosses the x -axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.

- Write an equation for the line tangent to the graph of g at $x = 1$.
- For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.
- The second derivative of g is $g''(x) = e^{f(x)}[(f'(x))^2 + f''(x)]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.
- Find the average rate of change of g' , the derivative of g , over the interval $[1, 3]$.

(a) Point: $g(1) = e^{f(1)}$
 $= e^2$ because $f(1) = 2$

SLOPE: $g(x) = e^{f(x)}$
 $g'(x) = f'(x)e^{f(x)}$
 $g'(1) = f'(1)[e^{f(1)}]$
 $= (-4)e^2$

$u = f(x)$
 $\frac{du}{dx} = f'(x)$

then the equation of the TAN line is

$$y - e^2 = -4e^2(x - 1)$$

(b) $g'(x) = f'(x) e^{f(x)}$ $e^{f(x)} > 0$ for all x
 $g'(x) = 0$ at $x = -1, 3$ $f'(x) = 0$ at $x = -1$ and 3
 $g'(x)$ changes from positive to negative values at $x = -1$ Hence $g(x)$ has a rel max at $x = -1$

(c) $g''(x) = e^{f(x)} [(f'(x))^2 + f''(x)]$
 $g''(-1) = e^{f(-1)} [(f'(-1))^2 + f''(-1)]$
 $e^{f(-1)} > 0$ $f'(-1) = 0$ $f''(-1) < 0$
 since f' is decreasing near $x = -1$

Hence $g''(-1) < 0$

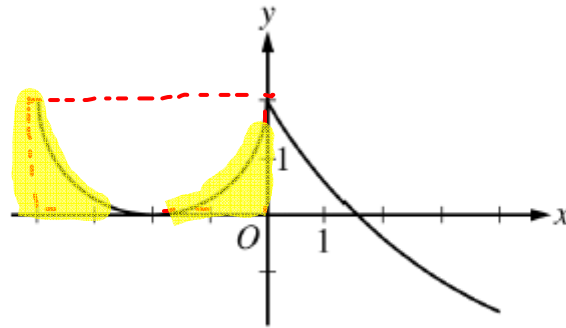
(d)

$$\begin{aligned} \text{AV Rate of } \Delta \text{ on } [1, 3] \text{ of } g' &= \frac{g'(3) - g'(1)}{3 - 1} \\ &= \frac{e^{f(3)} f'(3) - e^{f(1)} f'(1)}{3 - 1} \\ &= \frac{0 - (-4e^2)}{2} \end{aligned}$$

or $2e^2$
 The SIMPLIFIED VERSION

2009 AB6 [non-calculator]

AREA OF
RECTANGLE
- AREA OF
SEMI-CIRCLE
= YELLOW AREA



Graph of f'

The derivative of a function f is defined by

$$f'(x) = \begin{cases} g(x) & -4 \leq x \leq 0 \\ 5e^{-x/3} - 3 & 0 < x \leq 4 \end{cases}$$

The graph of the continuous function f' is shown above, has x-intercepts at $x = -2$ and $x = 3 \ln\left(\frac{5}{3}\right)$.

The graph of g on $-4 \leq x \leq 0$ is a semi-circle, and $f(0) = 5$

(a) For $-4 < x < 4$ find all value(s) of x at which the graph of f has a point of inflection

At $x = -2$ the graph of f' changes from decreasing to increasing

At $x = 0$ the graph of f' changes from increasing to decreasing

Hence f has p of I at $x = -2$ and $x = 0$

(b) Find $f(-4)$ and $f(4)$ -4

$$f(-4) = f(0) + \int_0^{-4} f'(x) dx$$

WHY THIS WORKS

$$= f(0) + [f(-4) - f(0)]$$

$$f(-4) = f(0) - \int_{-4}^0 f'(x) dx$$

$$= 5 - [8 - 2\pi] = 2\pi - 3$$

$$f(4) = f(0) + \int_0^4 f'(x) dx$$

$$= f(0) + \int_0^4 [5e^{-\frac{x}{3}} - 3] dx$$

$$= 5 + [-15e^{-\frac{x}{3}} - 3x] \Big|_0^4$$

$$= 5 + [(-15e^{-\frac{4}{3}} - 12) - (-15 - 0)]$$

(c) For the closed interval $-4 \leq x \leq 4$, find the value of x at which f has an absolute maximum.

CRITICAL VALUES at $x = -2$, $x = 3 \ln(\frac{5}{3})$
[$f'(x) = 0$]

at $x = 3 \ln(\frac{5}{3})$ f' changes from pos to neg
VALUES.

$f'(x) > 0$ on $-4 < x < -2$

f incr $f'(x) > 0$ on $-2 < x < 3 \ln(\frac{5}{3})$

decr $f'(x) < 0$ on $3 \ln\left(\frac{5}{3}\right) < x < 4$

$f(-4) = 2\pi - 3$ and f increases
on $-4 < x < -2$, $-2 < x < 3 \ln\frac{5}{3}$ [see above]
Then f decreases on $3 \ln\frac{5}{3} < x < 4$ [see above]
Hence f has an ab max at $x = 3 \ln\frac{5}{3}$

Concert ticket part d

$$r(t) = 500 e^{-\frac{t}{2}} \quad \text{tix/hr}$$

$$\int_0^3 r(t) dt$$
$$= \int_0^3 500 e^{-\frac{t}{2}} dt$$

$$\approx 777 \text{ tix}$$

will give us # of tix
sold between noon & 3pm

use your TI!

2008 AB2B [calculator-friendly]

For time $t \geq 0$, let $r(t) = 120(1 - e^{-10t^2})$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is

$$\text{modeled by } g(x) = 0.05x \left(1 - e^{\frac{-x}{2}} \right)$$

(a) How many kilometers does the car travel during the first 2 hours?

(b) Find the rate of change *with respect to time* of the number of liters of gasoline used by the car when $t = 2$. Indicate units of measure.

(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?