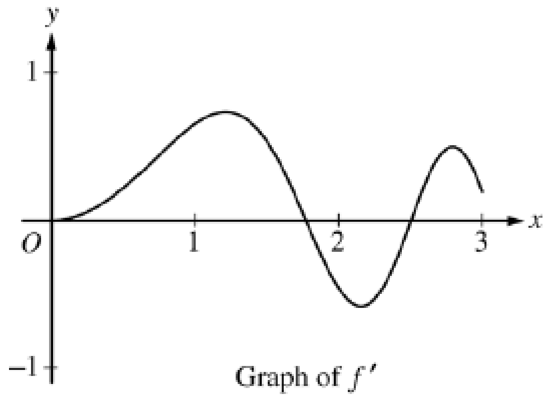
2006 AB2B [***calculator-friendly***]



Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{\frac{-x}{4}} \sin\left(x^2\right)$. The graph of $y = f'(x)$ is shown above.

(a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the open interval $1.7 < x < 1.9$. Justify.

The graph of $f'(x)$ is decreasing on $1.7 < x < 1.9$. Hence f is concave down on $1.7 < x < 1.9$

(b) On the closed interval $0 \le x \le 3$, find the value of x at which f has an ***absolute maximum.*** Justify.

At $x \approx 1.77245$ f' changes from positive to negative so at $x \approx 1.77245$ f has at least a rel max

Candidates for ab max: $x=0, x=3, x \approx 1.77245$

$$f(0) = 5$$

$$f(3) = f(0) + \int_0^3 f'(x)\,dx$$

$$\approx 5.57893$$

$$f(1.77245) = f(0) + \int_0^{1.77245} f'(x)\,dx$$

$$\approx 5.67911$$

Hence f has an ab max at $x \approx 1.77245$

```
          .57892848
Ans+5
         5.57892848
fnInt(Y1,X,0,A)
        .6791141303
Ans+5
         5.67911413
```

(c) Write an equation for line tangent to the graph of f at $x = 2$

To find the point:

$$f(2) = f(0) + \int_0^2 f'(x)\,dx$$

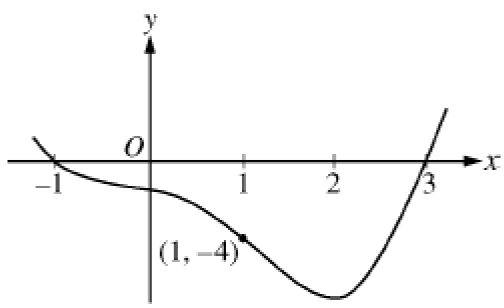
$$\approx 5.62342$$

$f'(2) \approx -0.45902$

$y - 5.62342 = -0.45\ 02(x - 2)$

```
          .6234267297
Ans+5
           5.62342673
Y1(A)
       6.42034356E-14
Y1(2)
         -.4590239168
█
```

2009 AB5B ***[non-calculator]***



Graph of f'

Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.

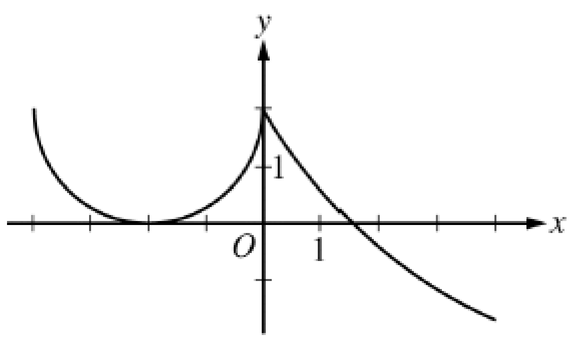
(a) Write an equation for the line tangent to the graph of g at $x = 1$.

(b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

(c) The second derivative of g is $g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.

(d) Find the average rate of change of g', the derivative of g, over the interval $[1, 3]$.

2009 AB6 ***[non-calculator]***



Graph of f'

The derivative of a function f is defined by

$$f'(x) = \begin{cases} g(x) & -4 \le x \le 0 \\ 5e^{-x/3} - 3 & 0, x \le 4 \end{cases}$$

The graph of the continuous function f' is shown above, has x-intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$.

The graph of g on $-4 \le x \le 0$ is a semi-circle, and $f(0) = 5$

(a) For $-4 < x < 4$ find all value(s) of x at which the graph of f has a point of inflection

(b) Find $f(-4)$ and $f(4)$

(c) For the closed interval $-4 \le x \le 4$, find the value of x at which f has an absolute maximum.

2008 AB2B [***calculator-friendly***]

For time $t \ge 0$, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x) = 0.05x\left(1 - e^{\frac{-x}{2}}\right)$

(a) How many kilometers does the car travel during the first 2 hours?

(b) Find the rate of change ***with respect to time*** of the number of liters of gasoline used by the car when $t = 2$. Indicate units of measure.

(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?